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Measurement of the Lifetime of Muons, Pions and Kaons

Master lab course

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Summary

In this lab course, muons, pions and kaons originated by the interaction of cosmic rays in the atmosphere are to be stopped and detected by a liquid scintillator. The lifetime of these particles is to be determined from their decay-time spectra.

What you will learn:

- The formation process of cosmic rays and how to detect them at sea level
- The working principle of scintillator detectors and photomultipliers
- How to handle a basic NIM module setup
- How to perform a simple data analysis with the ROOT framework

Contents

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Theoretical and Experimental Overview

1.1 Cosmic Rays

The existence of radiation passing through the atmosphere coming from outer space was demonstrated at the turn of the 19^{th} century by measurements on the conductivity of gases. At that time, it was believed that gases should be almost nearly perfect insulators provided that the applied electric field was not too high.

However, many scientists, among them C.T.R. Wilson, Elster and Geitel, showed that, in spite of careful precautions to prevent the known radiations from reaching the samples of air in their ionisation chambers, a significant residual conductivity remained. It was found that a reduction in conductivity resulted from shielding the ionisation chambers by lead and this fact was interpreted as showing that most of the residual conductivity was due to an external radiation of some form.

Hess and Kohlhörster investigated the conductivity and ionisation of the atmosphere in 1913. They found that the ionizing radiation increased with increasing altitude in the atmosphere. Therefore, they suggested that this radiation must be of extraterrestrial origin.

The development of cloud chambers and nuclear emulsions enabled the investigation of the relatively complicated interaction processes of cosmic rays. From the fifties on, the use of accelerators also helped to identify the various elementary particles and their characteristic interactions. These investigations resulted in the definition of the following components of cosmic rays: nucleons, muons, β- and γ-rays, and neutrinos.

One distinguishes between primary cosmic rays which are incident at the top of the atmosphere and secondary cosmic rays, produced by the propagation of primary cosmic rays through the atmosphere. Primary cosmic rays originate mainly from supernova explosions or pulsars. Another propagation mechanism was proposed by Fermi in 1949 and this describes the acceleration of interstellar matter by collisions with extended time dependent cosmic magnetic fields. Primary cosmic rays are mainly protons (80-90%), 10-15% are α-particles, i.e. helium nuclei and there is about 1% nuclei with $Z \geq 3$. In addition, primary electrons have an abundance of about 1%.

The energies of these primary cosmic rays span a range from a few eV to 10^{21} eV.

Particles which are capable to penetrate the Earth's magnetic field collide with nuclei of the atmosphere. Depending on the energy of the primary cosmic ray particle, more or less secondary particles or nuclear fragments are produced. These fragments are produced under relatively small transverse momenta and are sufficiently energetic to induce further reactions. This initiates a cascade like process that only stops if the energy of secondary particles is sufficiently low to fall below the threshold for further nuclear processes ($E < 1$ GeV).

Secondary cosmic radiation at sea level is subdivided into the soft ($e^{\pm}$, γ) and penetrating component, where the penetrating component essentially consists of muons with energies between 10^8 and 10^{12} eV. Muons are electromagnetic and weakly interacting particles. Their rest mass is approximately $207\,m_e$. They have a lifetime of 2,2 μs. The fact that muons are sufficiently energetic and decay only under weak interactions allows them reach sea level.

If one assumes that muons have a speed close to the speed of light, their average range based on classical mechanics would only be

$$s = c\tau = (3 \times 10^8\,\mathrm{ms}^{-1}) \cdot (2.2 \times 10^{-6}\,\mathrm{s}) \approx 700\,\mathrm{m} \quad .$$

Since muons are produced at an altitude of about 15 km, this would mean that practically no muon could be recorded at sea level. However, as a matter of fact, one observes a flux of about 1 particle per cm^2 per minute. This apparent contradiction is resolved if one considers that the lifetime of muons is dilated due to relativistic effects as further discussed in Sec. 1.2.

Pions and kaons are produced in the hadronic cascade and can also reach sea level.

The propagation of cosmic rays through the atmosphere is sketched in Fig. 1.1 and the altitude-dependent fluxes of protons, electrons and muons are shown in Fig. 1.2.

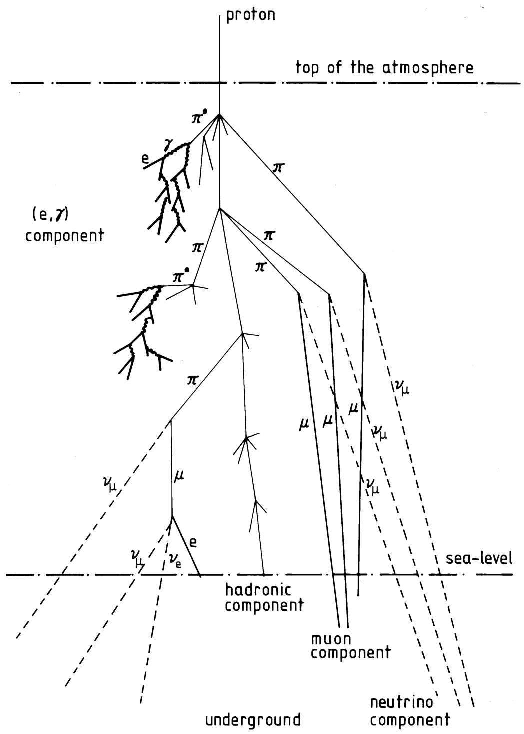

Figure 1.1: Propagation of cosmic rays in the atmosphere.

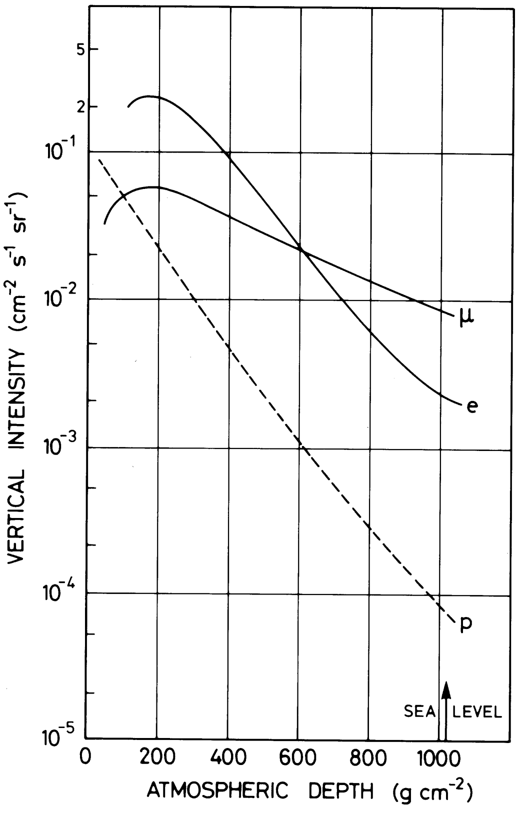

Figure 1.2: Altitude dependence of muon, proton and electron fluxes in the atmosphere.

1.2 Theory of Special Relativity

One of the two postulates of special relativity theory is that the speed of light in vacuum is the same for all inertial frames of reference. It is measured to be $c \approx 3 \cdot 10^8\,\mathrm{m/s}$. The transformation from one frame of reference to a different one moving at a velocity v is given by the Lorentz transformation. This introduces the so-called Lorentz factor γ, calculated as

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad . \tag{1.1}$$

It can be shown that the time lapse between two events in a certain frame of reference is different when viewed from a frame of reference moving at a velocity v with respect to the first frame. Namely,

$$\Delta t' = \gamma \Delta t \quad . \tag{1.2}$$

The time interval is longer in the moving frame and this result is called time dilation. The time interval measured in the frame where the clock is at rest is called the proper time.

Moreover, the relation between the momentum p of a particle and its total energy is given by

$$E = c\sqrt{p^2 + m_0^2 c^2} \quad . \tag{1.3}$$

This energy can be split into two components: the kinetic energy and the energy associated to its rest mass, $m_0 c^2$. The kinetic energy of a particle is then simply.

$$E_{\text{kin}} = E - m_0 c^2 \quad . \tag{1.4}$$

1.3 Physics of Muons, Pions and Kaons

The muon is an elementary particle and belongs to the second generation of leptons. Just as electrons they are subject to weak and electromagnetic interactions.

It was discovered by Anderson and Neddermeyer in 1937. Originally, the muon was suspected to be the particle postulated by Yukawa which was supposed to be responsible for the strong interactions. Only after the discovery of the pion in 1949, it became clear that pions and muons are different particles.

There exists negative and positive muons – particle and antiparticle – with charges $-1e$ and $+1e$, respectively. The dominant source of muons is pion decays, but muons are also produced in meson decays, such as kaons. Muons decay with an average lifetime of $\tau = 2.2\,\mu\mathrm{s}$ into an electron (e^+ or e^-) and two neutrinos. Since 1962, one knows that these two neutrinos are different particles. They are called the muon neutrino and electron neutrino (ν_μ and ν_e).

The muon decay obeys several conservation laws such as charge and lepton number conservations, where lepton numbers are independently conserved for the muon and the electron species. Energy, linear and angular momentum conservations also naturally hold.

	Rest mass (MeV/c^2)	Mean life (s)
$\mu^\pm$	$105.6583755 \pm 0.0000023$	$(2.1969811 \pm 0.0000022) \times 10^{-6}$
$\pi^\pm$	139.57039 ± 0.00017	$(2.6033 \pm 0.0005) \times 10^{-8}$
$K^\pm$	493.677 ± 0.013	$(1.2379 \pm 0.0021) \times 10^{-8}$

Table 1.1: Characteristic properties of muons, pions and kaons.[1]

Description	Process	Branching fraction
Muon decay	$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$	$\sim 100\%$
Muon capture	$\mu^- + p \to n + \nu_\mu$	–
Pion decay	$\pi^+ \to \mu^+ + \nu_\mu$ $\pi^- \to \mu^- + \bar{\nu}_\mu$	$\sim 99.99\%$
	$\pi^+ \to e^+ + \nu_e$ $\pi^- \to e^- + \bar{\nu}_e$	$\sim 0.01\%$
Kaon decay	$K^+ \to \mu^+\nu_\mu$ $K^- \to \mu^-\bar{\nu}_\mu$	$\sim 63.56\%$
	$K^\pm \to \pi^\pm\pi^0$	$\sim 20.67\%$

Table 1.2: Dominant decay processes of muons, pions and kaons.[1]

The decay processes of muons are three-body decays, which means that the spectrum of decay electrons are continuous in a similar way as the decay spectrum of electrons in nuclear beta decay. The maximum transferable energy to the electron is approximately half the rest mass of the muon corresponding to 53 MeV. The electron spectrum of a muon decay at rest is shown in Fig. 1.3.

In competition to muon decay, negative muons can also be captured by nuclei in a similar fashion as electrons from the K-shell can be captured by a nucleus and form a muonic atom. The probability for a muon capture increases with the fourth power of the nuclear charge number.

Since the rest mass of muons is roughly 200 times the one of electrons, their orbit is correspondingly 200 times closer to the nucleus. Consequently, there is a non-vanishing overlap of the muon wave function with the nuclear one and this can lead to a capture of the negative muon by the nucleus. As an example, for charge numbers $Z \geq 40$, the Bohr orbit is already inside the nucleus and the capture probability in this case is very close to 1. For light nuclei, however, with $Z \lesssim 10$, negative muons will preferably decay as their positive counterparts.

The decay probability for free muons at rest $\lambda = \frac{1}{\tau}$ is the same for positive and negative muons.

$$\lambda^+ = \lambda^- \qquad \text{or} \qquad \tau^+ = \tau^-$$

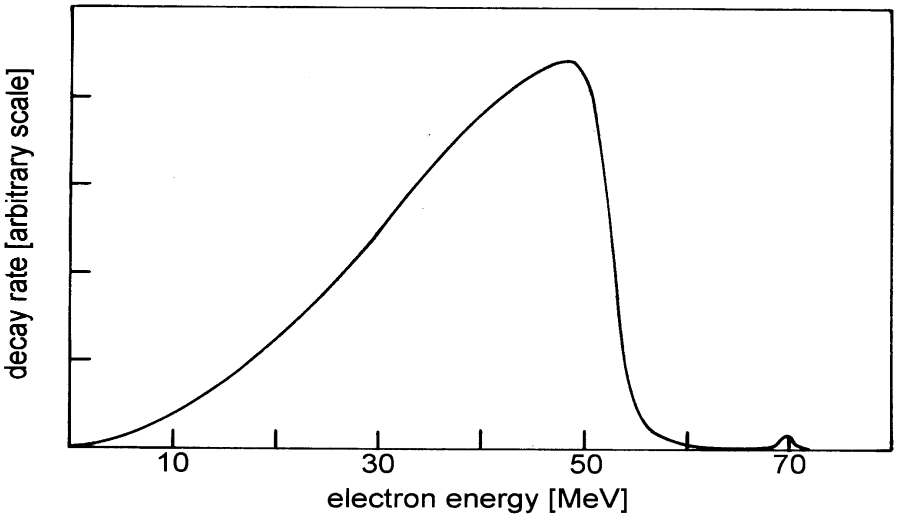

Figure 1.3: Energy spectrum of electrons from muon decay at rest.

If negative muons are stopped in matter, the measured decay probability λ_m^- is always larger than the decay probability λ^+ or λ^- for free muons. So, the equation

$$\lambda_m^- > \lambda^+ = \lambda^-$$

holds or, respectively

$$\tau_m^- < \tau^+ = \tau^- \quad .$$

In this experiment, muons are stopped in a liquid scintillator. The scintillation material consists essentially of carbon. The mean life of negative muons in carbon has been measured to be roughly $2.026\,\mu$s. For lighter elements, e.g. hydrogen or helium, the mean life is equal to the lifetime of a free muon ($\approx 2.20\,\mu$s).

1.4 Scintillators and Photomultipliers

The liquid scintillator mixture used in this experiment is composed of three different components: the primary scintillator, the wavelength shifter and the base material. Certain types of molecules will release a small fraction of the absorbed energy as optical photons. This process is especially marked in organic substances which contain aromatic rings, such as polystyrene, polyvinyl toluene and naphthalene. Liquids which scintillate include toluene or xylene.

This primary scintillation light is preferentially emitted in the UV range. The absorption length for UV-photons in the scintillation material is rather short and the scintillator is not transparent for its own scintillation light. Therefore, this light is transferred to a wavelength shifter which absorbs the UV light and re-emits it at a longer wavelength (e.g. in the green). Due to the lower concentration of the wavelength-shifter material, the emitted light can traverse the scintillator and be detected by a photomultiplier. The technique of wavelength shifting is also used to match the emitted light to the spectral sensitivity of the photomultiplier. For plastic scintillators, the primary scintillator and wavelength shifter are mixed with organic material to form a polymerizing structure, whereas in liquid scintillators, the two active components are mixed with an organic base. About 100 eV are required to produce one photon in an organic scintillator.

In this experiment, the liquid scintillator consists of *p*-terphenyl as the primary scintillator, POPOP as the wavelength shifter and Uvasol as the base material. The readout is accomplished by four photomultiplier tubes that are coupled to the liquid scintillator via an air light guide.

1.5 Determination of the Mean Life

The law of radioactive decay

$$N(t) = N_0 \, \mathrm{e}^{-\lambda t} \tag{1.5}$$

is a statistical law, which means that it is valid for many different decays. There is no way to predict the decay of an individual particle. In particular, the decay probability does not depend on the fact that a certain particle may have lived already for a certain time. The measurement of the mean life of muons and pions is based on the following principle.

The deceleration of a charged particle in the liquid scintillator gives rise to a light signal that is recorded by the photomultipliers. This is used as a trigger for the time-to-amplitude converter (TAC). When the particle has come to rest, no further light is emitted. Only the decay products that appear after decay provide a second signal at a later time which stops the TAC. The time difference between the incoming, e.g. stopping muon and the electron appearance consists of a measurement for the decay time of individual muons. By summing up many events of the same kind, one obtains a time spectrum of decay times which can be parametrized by the law of radioactive decay (Eq. 1.5).

If the incoming particles decay during flight, their deceleration signal cannot be distinguished from the one generated by their decay products and consequently, there will be no stopping input for the TAC. Therefore, in our case, for example, muons decaying in flight are simply ignored by the circuitry. Signals caused by radioactive background are rare and can be suppressed by properly adjusting the discriminator threshold, as will be further explained in Chap. 2.

Since muons in cosmic rays are a mixture of positive and negative muons, one will observe three different components in the time spectrum as measured by the TAC: the decay of free positive muons, the decay of negative muons and chance coincidences.

Chance coincidences can be measured at very large decay-time ranges where practically all muons have decayed. The contribution of chance coincidences is usually small but they must be taken into account. They are uniformly distributed over all times.

Since the decaying positive and negative muons have a different lifetime due to muon capture, a more accurate fit function would be the sum of two exponentials with individual lifetimes given the sum of two exponentials with different slopes is not an exponential itself. As mentioned in Sec 1.3, the lifetime of the negative muons can be fixed to 2.026 μs, the value for stopping negative muons in carbon.

Moreover, we can make use of the fact that there is an excess of positive muons at sea level, the ratio between positive and negative muons is constant over a wide momentum range and takes a value of

$$R = \frac{N_{\mu^+}}{N_{\mu^-}} \approx 1.27 \quad . \tag{1.6}$$

Details about the fitting function can be found in Chap. 3.

In a similar fashion, the spectrum recorded for pion decays contains four different contributions. The first contribution comes from the decays of kaons with a lifetime of 12.4 ns. The kaons dominate the spectrum at short decay-times ranges. The second contribution comes from chance coincidences which amounts to a uniform distribution. The third distribution comes from muon decays which is also rather uniform over a time range of 200 ns given muon lifetimes are rather long compared to the lifetime of pions. For this correction, one can use an average effective common lifetime for μ^+ and μ^-. Lastly, the fourth contribution comes from π^+ decay only, as π^- are always captured by nuclei. In summary, when analysing the pion decay spectrum one has to take into account chance coincidences, muon decays, and kaon decays.

Electronic noise affects the measurement at very short decay times ($\lesssim$ 45 ns), thus this part of the spectrum should not be used for the fitting process. See Chap. 3 for details.

2

Experimental Procedure

The light from the liquid scintillator is measured with four photomultipliers. The negative high voltage connected to the photomultipliers should never exceed ≈ -1.9 kV.

The start and stop signals can be derived from a majority condition. In principle, the majority condition depends on the background, the chosen discriminator thresholds and the high voltage. In pion decays, muons get only very little kinetic energy so that a soft majority condition should be used for the measurement of pion decays. Since the electrons in muon decays can get a rather large kinetic energy, a more stringent majority condition could be selected to reject background. However, it has been found that a majority condition of “2 out of 4” gives good results in both cases.

The circuit diagram for the measurement of the mean life of muons and pions is shown in Fig. 2.1. The gate generator defines the time window in which stop signals can be accepted. In the same way, the gate generator inhibits start signals which occur during the time window.

1. Observe the signals of the photomultpliers on the oscilloscope and sketch their signal shape.
2. Observe the logic signals after the discriminators and adjust the threshold of the discriminators in such a way that muons and pions and their decay products can be measured.
3. Check the wiring of all NIM modules and also check the proper functioning of the circuit by looking at the signals in different places using the oscilloscope. Use the pulse generator which supplies signals of known width and known relative delay. Make a sketch of all signal shapes at all important points of the electronic circuitry.
4. For the measurement of muons one should select a time window on the gate generator of the order of $10\,\mu$s while for the measurement of pions the time window should be of the order of $200\,$ns.
5. Describe and motivate the electronic logic for the measurement of the decay times of muons and pions.

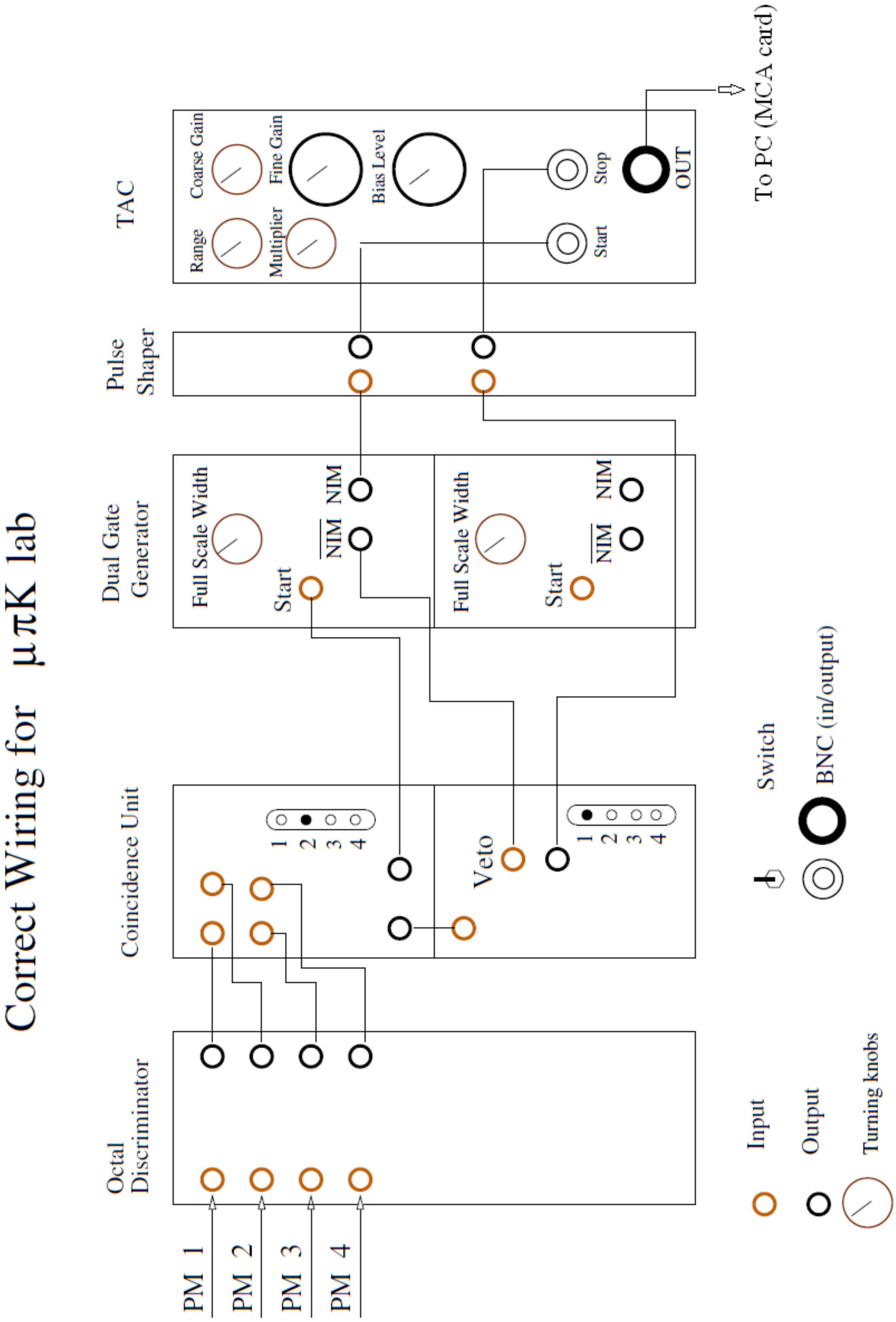

Figure 2.1: Circuit diagram for the lifetime measurement.

3

Data Analysis

The data are recorded by a multichannel analyser and can be graphically displayed on a screen.

1. **Determine the lifetime of muons**. The recorded data are to be corrected for instrumental effects like background.

   (a) Start with the simple fit function

   $$N_1(t) = N_0\,\mathrm{e}^{-t/\tau} + b \tag{3.1}$$

   assuming that positive and negative muons have the same lifetime.

   (b) Then use the reduced lifetime of stopping negative muons (due to muon capture) and the ratio of negatively and positively charged muons as given in Eq. 1.6 for a modified fit function

   $$N_2(t) = N_0(\mathrm{e}^{-t/\tau_{\mu^+}} + (1/1.27)\,\mathrm{e}^{-t/2.026}) + b \tag{3.2}$$

   Compare and discuss the two results.

2. **Determine the lifetime of pions and kaons**. The decay spectrum of pions has to be corrected for chance coincidences and kaon and muon decays.

   (a) Determine the pion mean life from a fit

   $$N_3(t) = N_0\,\mathrm{e}^{-t/\tau_\pi} + b \tag{3.3}$$

   to the data for time differences where one can be sure that the kaon decay can be neglected (e.g. larger than ≈ 70 ns). The contributions from chance coincidences and muon decay are taken as a uniform background b.

(b) The lifetime of the kaons is so short that it cannot be determined very precisely with our experimental setup. One can make an exponential fit like for the pions in a small time interval (45 - 60 ns). Another option is to use a double exponential fit function

$$N_4(t) = N_{0_K}\,\mathrm{e}^{-t/\tau_K} + N_{0_\pi}\,\mathrm{e}^{-t/\tau_\pi} + b \tag{3.4}$$

with independent parameters for kaons and pions and fit over the full time range (excluding very short decay times < 45 ns) .

Compare and discuss the results of the different fitting functions.

3. **Determine statistical and systematic errors.** Check how variations of the lower and upper fit-limits change the results of the fit.

4. **Plot all data in a graphic way on both linear and semi-logarithmic scales.**

5. **Determine the rate of muons and the rate of pions which stop in the detector**. To estimate the numbers of muons and pions, use the fit function $N_1(t)$. The area under the exponential function gives a good estimate for the number of particles. Integrating the exponential function from 0 to infinity gives $N_{\mathrm{tot}} = N_0\tau$. Now one just needs to divide this number by the slope of the calibration, since the spectrum was derived in channels, to get the number of muons or pions.

4

Problems

1. Determine the minimum kinetic energy of a muon produced at an altitude of 20 km so it reaches sea level. Feel free to use the following approximations for the muon mean life and rest mass, respectively: $\tau_\mu = 2.2\,\mu\text{s}$ and $m_\mu = 105.7$ MeV/c^2.
2. Determine the kinetic energy of a muon originated from the decay of a pion at rest. Feel free to use the following approximation for the pion rest mass: $m_\pi = 139.6$ MeV/c^2.
3. Explain the small bump at 70 MeV in Fig. 1.3.

Appendix

A

Operation Instructions for the Multi-Channel Analyser

The measurement is done using a Multichannel Analyser Card ‘MCA-3A’ installed in a personal computer running Windows 10.

The provided ‘MCDWIN’ software allows to produce a linear or logarithmic plot of the time spectrum, perform Gaussian fits to the calibration spectrum, select data intervals using the mouse, and so on. The supervisor will show you how to do the measurements using the ‘MCDWIN’ software. Details of the software are provided in the manual
https://www.fastcomtec.com/ftp/manuals/mca3doc.pdf

After the data taking is complete, the data file is saved on the lab computer and can be transferred to an USB-Stick in ASCII format, so the analysis can be done with your home computer.

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