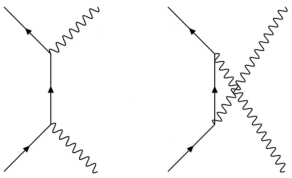
Computer-based calculation of Born-level QED processes

PD Dr. Tobias Huber
Email: huber@physik.uni-siegen.de

October 6, 2021



Abstract

In this project, students get – on a practical level – familiar with the series of steps required to perform the full calculation of a scattering process in quantum electrodynamics (QED) in the Born approximation ("tree level").

The steps include the derivation of Feynman rules from a Lagrangian, generation of the diagrams and the corresponding amplitude, squaring of the amplitude and calculation of the cross section. For all tasks a computer-based approach is envisaged, based on the programming language `Mathematica`.

As a prerequisite the students should have attended "Theoretical Particle Physics 1" or "Concepts and phenomena of particle physics". Also basic knowledge about a computer algebra program such as `Mathematica` is beneficial.

Prior to starting this project

- Contact the supervisor about access to `Mathematica`.

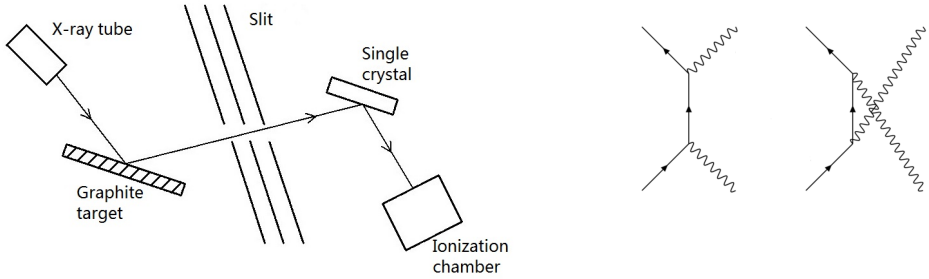


Figure 1: Left: Schematic diagram of Compton's experiment (from Wikipedia). Right: Tree-level diagrams for Compton scattering $e^-\gamma \to e^-\gamma$.

1 Introduction

Elementary processes in quantum electrodynamics (QED) such as Bhabha scattering ($e^+e^- \to e^+e^-$), Møller scattering ($e^-e^- \to e^-e^-$), Compton scattering ($e^-\gamma \to e^-\gamma$), pair annihilation ($e^-e^+ \to \gamma\gamma$), and muon pair creation ($e^+e^- \to \mu^+\mu^-$) have played a major rôle since the early days of particle collision experiments. The scattering experiment by Compton (fig. 1), for instance, supplied evidence that photons carry momentum, and that they can transfer momentum to other particles. Moreover, Bhabha scattering (fig. 2) is among the prime processes to measure the luminosity at e^+e^- colliders, which in turn is an indispensable input when measuring cross sections of basically any other process.

On the theoretical side, the technical steps that are required to carry out the calculation of a cross section are the derivation of the Feynman rules from the Lagrangian density, the generation of the Feynman diagrams and the transition amplitude, the squaring of the latter, and the integration over the final-state phase space. Since all these steps follow more or less straightforwardly from well-established rules in quantum field theory, there is a high potential of automation. Therefore, the computer has become an indispensable tool in carrying out calculations in quantum field theory. Many dedicated programs and packages were developed, tailored for particular steps of the calculation or for carrying out the calculation as a whole.

In this project, we first get familiar with the theory of quantum electrodynamics (QED), and repeat the basic steps that have to be performed in a cross-section calculation. We will then briefly introduce the `Mathematica` packages `FeynRules`, `FeynArts`, and `FeynCalc`, which allow to automate many steps of the calculation. The task will then be to use these packages to compute in QED the cross sections of the muon pair production and the compton scattering process in the Born approximation (i.e. at "tree-level").

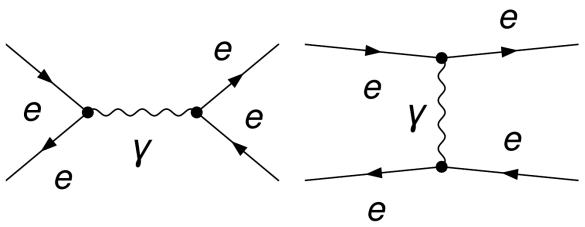


Figure 2: Tree-level Feynman diagrams for BhaBha scattering $e^+e^- \to e^+e^-$.

2 Quantum electrodynamics

Quantum electrodynamics (QED) is a relativistic, quantum-field-theoretic description of the electromagnetic interaction of electrons and positrons (charged Dirac fermions in general) with photons.

2.1 Lagrangian density, gauge symmetry

The Lagrangian density of QED reads

$$\mathcal{L} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free photon field}} + \underbrace{\bar{\psi}(i\partial\!\!\!/ - m)\psi}_{\text{free electron field}} + \underbrace{e\,\bar{\psi}A\!\!\!/\psi}_{\text{interaction}} \,.$$

Here ψ is a Dirac spinor describing the fermion (electron, positron), A_μ is a vector field describing the photon, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor. m is the fermion mass, and $e = \sqrt{4\pi\alpha}$ the electric charge of the positron ($\alpha \sim 1/137$ is the fine structure constant). In $\mathcal{L}$ we distinguish terms that describe free fields from interaction terms. The former contain at most two fields, and are classified as mass terms (with masses) or kinetic terms (with derivatives). The interaction terms have more than two fields.

The Lagrangian density of QED is invariant under the local gauge transformation

$$\psi(x) \to \psi'(x) = e^{ie\Lambda(x)}\,\psi(x)\,,$$
$$A_\mu \to A'_\mu(x) = A_\mu + \partial_\mu\Lambda(x)\,.$$

Moreover, the interaction term can be obtained from the theory of free fields by means of a procedure called "minimal substitution",

$$\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu(x)\,.$$

D_μ is called the "covariant derivative".

2.2 Feynman rules

The Feynman rules that can be derived from the Lagrangian density are classified as propagators, external lines and vertices, and are usually given in momentum space.

The propagators are the Green's functions of the free field equations ($e = 0$), e.g. for the electron

$$(i\not{\partial} - m)G(x) = \delta^{(4)}(x)\,,$$

which after Fourier transformation

$$G(x) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ipx}\, \tilde{G}(p)\,, \qquad \delta^{(4)}(x) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ipx}$$

gives

$$\tilde{G}(p) = \frac{\not{p} + m}{p^2 - m^2 + i\eta}\,.$$

The electron propagator is then $\tilde{S}(p) = i\,\tilde{G}(p)$, and the infinitesimal $\eta > 0$ moves the poles away from the real axis such as to have causal boundary conditions. The attempt to perform analogous steps to derive the photon propagator $D^{\nu\rho}(x) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ipx}\, \tilde{D}^{\nu\rho}(p)$ from

$$(-g_{\mu\nu}\Box + \partial_\mu\partial_\nu)D^{\nu\rho}(x) = -\delta^\rho_\mu\, \delta^{(4)}(x)$$

fails since the equation

$$(p^2 g_{\mu\nu} - p_\mu p_\nu)\tilde{D}^{\nu\rho}(p) = -\delta^\rho_\mu$$

cannot be inverted ($M_{\mu\nu} = p^2 g_{\mu\nu} - p_\mu p_\nu$ has an eigenvalue zero due to $p^\mu M_{\mu\nu} = 0$). One possible solution is to add a gauge-fixing term to $\mathcal{L}$,

$$\mathcal{L} \to \mathcal{L} - \frac{1}{2\xi}\,(\partial_\mu A^\mu)^2\,,$$

which then gives

$$\tilde{D}^{\nu\rho}(p) = -\frac{g^{\nu\rho} - (1-\xi)\frac{p^\nu p^\rho}{p^2+i\eta}}{p^2 + i\eta}\,.$$

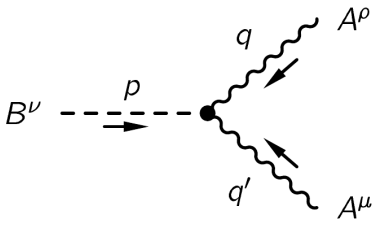
The photon propagator is then $\tilde{\Delta}^{\nu\rho}(p) = i\,\tilde{D}^{\nu\rho}(p)$. ξ is called *gauge parameter*. Physical observables do not depend on ξ. One of the common choices is the *Feynman gauge* $\xi = 1$ which we will use throughout.

The Feynman rules for the external lines can simply be taken from the classical solution of the free field equations. For the electron/positron we get the familiar spinors $u(p,s)$, $\bar{u}(p,s)$, $v(p,s)$, $\bar{v}(p,s)$ for incoming/outgoing (anti)fermion. For the photon we obtain the polarization vector $\epsilon^\mu(p,\lambda)$.

In order to derive Feynman rules for interaction vertices from $\mathcal{L}$ we first give the general prescription, together with an example, and subsequently apply it to QED. The recipe consists of four steps

1. Search for all products in $\mathcal{L}$ containing a previously specified combination of fields. These form the external lines of the vertex.

   E.g. $-g(\partial_\mu A_\nu)A^\mu B^\nu = -g(\partial_\mu A^\rho)g_{\rho\nu}A^\mu B^\nu$



2. Replace all derivatives by $(-i)$ times the incoming momenta of the fields on which they act.

   $\longrightarrow \quad -g(-iq_\mu)g_{\rho\nu}A^\rho A^\mu B^\nu = ig(q_\mu g_{\rho\nu})A^\rho A^\mu B^\nu$

3. Sum over all permutations of the indices and momenta of identical external fields. (Note that ψ and $\bar{\psi}$ are not identical fields!)

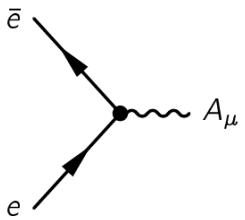
   $\longrightarrow \quad ig(q_\mu g_{\rho\nu} + q'_\rho g_{\mu\nu})A^\rho A^\mu B^\nu$

4. Discard all external fields.

   $\longrightarrow \quad ig(q_\mu g_{\rho\nu} + q'_\rho g_{\mu\nu})$.

Applied to QED we find

1. $ie\bar{\psi}\gamma_\mu\psi A^\mu$
2. $ie\bar{\psi}\gamma_\mu\psi A^\mu$
3. $ie\bar{\psi}\gamma_\mu\psi A^\mu$
4. $ie\gamma_\mu$.



These rules are supplemented by the following items

- Impose four-momentum conservation at each vertex
- Integrate over each unconstrained (loop) momentum with measure $\displaystyle\int \frac{d^4k}{(2\pi)^4}$

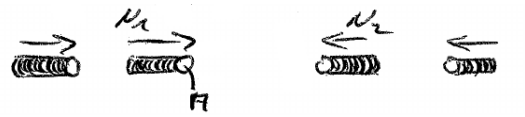
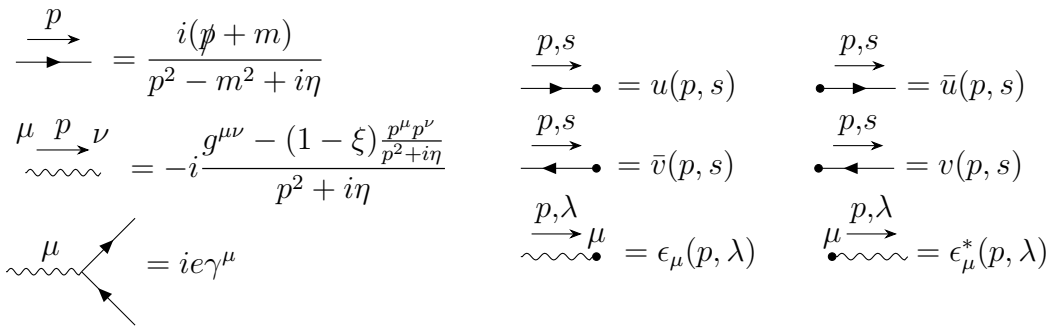


Figure 3: Bunches in a storage ring experiment.

- Along a fermion line, order the Dirac matrices and spinors opposite to the direction of fermion flow.
- Include a factor (-1) and a Dirac trace for each closed fermion loop.
- Include a factor (-1) when permutating external fermion lines in processes with more than one diagram.

More on Feynman rules in QED and other theories can be found in the literature on quantum field theory, e.g. [1–5]). We summarize the QED Feynman rules below.

2.3 QED Feynman rules, summary



3 Computing cross sections

One of the most important quantities at particle colliders is the differential cross section. To define it, we first have a look at another quantity, the *luminosity*, which is a measure of the density and velocity of particles and related to the particle flux *flux* in a beam. The flux ϕ is the number of particles N per time t that cross a certain area A,

$$\phi = \frac{N}{A\,t} = \frac{N\,v}{A\,l} = \frac{N\,v}{V} = \rho\,v\,,$$

where we have used that the velocity $v = l/t$, the volume $V = A\,l$, and the density $\rho = N/V$. For a storage ring experiment, where two oppositely

moving beams are brought to collision, the luminosity L is given by the formula

$$L = f\, n\, \frac{N_1\, N_2}{A}\,.$$

Here $N_{1,2}$ is the number of particles in each bunch, see figure 3, n is the number of bunches in one beam, and f the revolution frequeny. L is measured in units of $\mathrm{cm}^{-2}\mathrm{s}^{-1}$. Another important collider parameter is the *integrated luminosity*

$$\int dt\, L$$

which is a measure of the amount of data taken. Its unit $1/\mathrm{cm}^2$ is that of an inverse area, and it is usually expressed in inverse barns, b^{-1}, where $1\mathrm{b} = 10^{-24}$ cm.

The *differential cross section* $d\sigma$ is then defined as

$$d\sigma = \left(\frac{dN}{dt\, dPS}\right) dPS\, \frac{1}{L}\,,$$

where $dN/(dt\, dPS)$ is the number N of events per time t in a specific phase space element dPS. The phase space element can for instance be the solid angle, $dPS = d\Omega$, or the energy or (transverse) momentum of a particle. Expressed in terms of the Lorentz-invariant transition amplitude ("matrix element") $\mathcal{M}$, the formula for the differential cross section of the particle reaction $A + B \to F_1 + \ldots + F_n$ with momenta $k_1 + k_2 \to p_1 + \ldots + p_n$ reads

$$d\sigma = \underbrace{\frac{1}{4\sqrt{(k_1 k_2)^2 - m_A^2 m_B^2}}}_{\text{flux factor}} \underbrace{(2\pi)^4\, \delta^{(4)}(k_1 + k_2 - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \widetilde{dp}_i}_{\text{n particle phase space } R_n} \sum{}' |\mathcal{M}|^2\,.$$

We emphasize that the first two elements, the flux factor and the n particle phase space, are process-independent, and only the matrix element term is specific to the process. Here $\widetilde{dp} = d^3\vec{p}/(2\pi)^3/(2E_p)$ is the Lorentz-invariant integration measure for each final-state particle, and $\sum'$ denotes the following procedure. If the beams are unpolarized, we *average* over the all spins in the initial state (sum over the spins and divide by the number of physical spin projections). If the spins of the particles in the final state are not measured, we *sum* over the all spins in the final state.

The procedure for obtaining $i\mathcal{M}$ should be familiar from the particle physics lectures, a brief summary is

1. Draw all connected, amputated Feynman diagrams which are relevant to the process up to a given perturbative order.

2. Write down the mathematical expression that corresponds to each diagram following, e.g. for QED, the rules from the previous section.

3. Divide each diagram by its symmetry factor (see e.g. Peskin [2]).

4. Sum up all diagrams.

For computing the cross section, we must square the amplitude. When dealing with fermion lines, the following relations are useful. Let Γ be any string of Dirac matrices and $\bar{w}(p,s) = w^\dagger(p,s)\gamma^0$ for $w = u$ (fermions) or $w = v$ (anti-fermions). Then the relation

$$[\bar{w}_1(p,s)\,\Gamma\,w_2(p',s')]^* = \bar{w}_2(p',s')\,\bar{\Gamma}\,w_1(p,s)$$

with $\bar{\Gamma} = \gamma^0\,\Gamma^\dagger\,\gamma^0$ holds. Moreover,

$$\begin{aligned} \bar{\gamma}_\mu &= \gamma_\mu\,, & \overline{\gamma_\mu\gamma_5} &= \gamma_\mu\gamma_5\,, & \overline{\sigma_{\mu\nu}} &= \sigma_{\mu\nu}\,. \\ \bar{\gamma}_5 &= -\gamma_5\,, & \overline{\gamma_\mu\gamma_\nu} &= \gamma_\nu\gamma_\mu\,, & \overline{\Gamma_1\Gamma_2} &= \bar{\Gamma}_2\,\bar{\Gamma}_1\,. \end{aligned}$$

The squared amplitude can be turned into a Dirac trace by means of the so-called Casimir trick,

$$\begin{aligned} \left|\bar{w}_1(p,s)\,\Gamma\,w_2(p',s')\right|^2 &= \bar{w}_1(p,s)\,\Gamma\,w_2(p',s')\bar{w}_2(p',s')\,\bar{\Gamma}\,w_1(p,s) \\ &= \mathrm{Tr}\left[w_1(p,s)\bar{w}_1(p,s)\,\Gamma\,w_2(p',s')\bar{w}_2(p',s')\,\bar{\Gamma}\right]\,. \end{aligned}$$

The combinations $w\bar{w}$ are then turned into Dirac matrices via the formulas for the spin sums, (those for the photon polarization is given for completeness)

$$\sum_s u(p,s)\bar{u}(p,s) = \not{p} + m\,, \qquad \sum_s v(p,s)\bar{v}(p,s) = \not{p} - m\,,$$

$$\sum_\lambda \epsilon^{*\,\mu}(k,\lambda)\,\epsilon^\nu(k,\lambda) \to -g^{\mu\nu}\,.$$

Finally, traces of Dirac matrices obey

$$\begin{aligned} \mathrm{Tr}\left[\gamma_{\mu_1}\cdot\ldots\cdot\gamma_{\mu_{2n+1}}\right] &= 0 \qquad\qquad n \in \mathbb{N} \\ \mathrm{Tr}\left[\gamma_\mu\gamma_\nu\right] &= 4\,g_{\mu\nu} \\ \mathrm{Tr}\left[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\right] &= 4\,(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}) \\ \mathrm{Tr}\left[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\right] &= -4i\epsilon^{\mu\nu\rho\sigma}\,. \end{aligned}$$

For a $2 \to 2$ process $A + B \to F_1 + F_2$ the cross section differential in the center-of-mass solid angle $d\Omega_{\text{C.M.S.}}$ assumes the form

$$\frac{d\sigma(AB \to CD)}{d\Omega_{\text{C.M.S.}}} = \frac{1}{64\pi^2\,s}\,\frac{\lambda_{F_1F_2}(s)}{\lambda_{AB}(s)}\,\sum{}'\,|\mathcal{M}|^2$$

with the usual Mandelstam variable $s = (k_1 + k_2)^2$ and
$\lambda_{ij}(s) = \sqrt{(s - (m_i + m_j)^2)(s - (m_i - m_j)^2)}$.

The *total cross section* σ is the differential cross section integrated over the entire phase space,

$$\sigma = \frac{1}{S!} \int d\sigma.$$

Its unit is that of an area, which is usually given in barns. The factor $1/S!$ is a symmetry factor which stands for the following procedure. For every j identical particles in the final state, add a factor of $1/j!$.

4 The Mathematica package FeynRules

The derivation of Feynman rules from a Lagrangian follows straightforward rules, see the previous chapter, and is therefore highly suitable for automation. The package `FeynRules` [6–9] is dedicated to this task. Its manual can be found at http://feynrules.irmp.ucl.ac.be/ together with the package files. We use an example to illustrate the relevant commands. The package is shipped with a first example in the subdirectory Models/FirstExample/. It contains the model file FirstExample.fr for QCD (quantum chromodynamics) with three quark flavours. It contains the following definitions

- Index definitions, gives the types and ranges of indices, e.g.

```
IndexRange[ Index[Generation] ] = Range[3]
IndexRange[ Index[Colour] ] = Range[3]
IndexRange[ Index[Gluon] ] = Range[8]
IndexStyle[Colour, i]
IndexStyle[Gluon, a]
IndexStyle[Generation, f]
```

- Parameter list, in this case defines the coupling constant,

```
M$Parameters = {
gs == {InteractionOrder -> {QCD, 1},
Value -> 1.2, TeX -> Subscript[g, s]} }
```

- Gauge group list

```
M$GaugeGroups = { SU3C == { Abelian -> False,
GaugeBoson -> G, StructureConstant -> f,
```

```
Representations -> {T, Colour},
CouplingConstant -> gs}}
```

- Particle classes list. The model file contains two particle classes. First, the class `F[1]`, a fermion represented by a symbol `q`, containing three members (u, c, t). Second, the class `V[1]`, a vector represented by the symbol `G`. For instance for `V[1]` (the gluon)

```
M$ClassesDescription = { V[1] == { ClassName -> G,
SelfConjugate -> True, Indices -> {Index[Gluon]},
Mass -> 0, Width -> 0} }
```

- The Lagrangian is then entered in the usual textbook way

```
L = -1/4 FS[G, μ, ν, a] FS[G, μ, ν, a]
+ I HC[q].Ga[μ].DC[q, μ]
- MQ[f] HC[q[s, f, i]].q[s, f, i] .
```

  The meaning of the symbols is the following: `FS[G, μ, ν, a]` is the field strength tensor connected with `G`. `q[f,s,i]` is the quark field with Dirac index `s`, generation index `f` and colour index `i`, `HC` its hermitian conjugate. `MQ[f]` is the quark mass, `Ga[μ]` the Dirac matrix γ^μ, and `DC` the covariant derivative.

- With the Lagrangian at hand, several commands can be applied, for instance

  `ExpandIndices[L]` expands out the indices of the Lagrangian. Possible options include `FlavorExpand` (whether and which flavour indices should be expanded, values `True`, `False`, `Generation`. Moreover, there are `MaxParticles`, `MinParticles`, `MaxCanonicalDimension`, and `MinCanonicalDimension`, which select only those terms in the lagrangian with at most/at least the specified number of particles / the specified canonical dimension. Finally, `SelectParticles` selects only those terms in the lagrangian which contain exactly the specified external fields, for instance

```
ExpandIndices[L, SelectParticles -> {{G, G, G}, {G, G,
G, G}}]
```

  selects only the three- and four-gluon interaction terms.

- The Feynman rules can be obtained via

```
FeynmanRules[L];
```

which is one of the main commands of the package. The options `MaxParticles`, `MinParticles`, and `SelectParticles` can be applied here as well. Moreover, there are the options `Free` and `Contains`, as in

```
FeynmanRules[L, Free -> q];
FeynmanRules[L, Contains -> q];
```

5 The Mathematica package FeynArts

The package `FeynArts` [10] is a package to generate the diagrams and the transition amplitude of a particle reaction. Moreover, it has some nice graphical features which we explore below. Let us describe the main commands,

- One first has to generate the topology, essentially meaning the number of incoming and outgoing particles, and the number of loops.

  `CreateTopologies[l, i -> o, ExcludeTopologies -> Internal]`

  Creates all topologies with l loops (zero for tree-level), i incoming and o outgoing particles. The option "ExcludeTopologies → Internal" makes the program keep only one-particle irreducible (1PI) graphs.

- `FeynArts` distinguishes three levels of fields: Generic (`F`, `S`, `V`, `U`, `T`, `SV` for fermion, scalar, vector, ghost, tensor, scalar-vector mixing), Classes (fields with common properties, e.g. under charge conjugation), Particles (class members, antiparticles get a minus-sign in front of the field, e.g. `F[2,{1}]` is the electron, `-F[2,{1}]` is the positron. For inserting fields into topologies, we type for instance for Bhabha scattering

  `InsertFields[`*topo*`, {F[2, {1}], -F[2, {1}]} -> {F[2, {1}], -F[2, {1}]}]`

  Possible options include `InsertionLevel`, which can take one or more of the following, `{Generic}`, `{Classes}`, `{Particles}`. The option `ExcludeParticles` keeps only diagrams without the particles specified. `LastSelections` keeps only diagrams with the particles specified.

- `Paint[`*expr*`, ColumnsXRows -> `n`]` draws the set of diagrams with n graphs per line.

To make a specific example, the following series of commands generates the tree-level diagrams for Bhabha scattering shown in fig. 2,

```
bhabha = CreateTopologies[0, 2 -> 2];
InsertFields[bhabha, F[2, 1], -F[2, 1] -> F[2, 1], -F[2,
1], InsertionLevel -> Classes, ExcludeParticles -> S[1],
S[2], V[2], F[2, 2], F[2, 3]];
Paint[%];
Export["bhabha.pdf", %]
```

The last command saves the picture to a file. Other output formats such as .ps, .eps or .tex are also possible.

- Finally we create the amplitude with

  `CreateFeynAmp[`*diags*`]` ,

  where *diags* is the expression created with the `InsertFields` command.

Further commands can be found in the `FeynArts` guide, available at www.feynarts.de.

6 The Mathematica package FeynCalc

`FeynCalc` is a `Mathematica` package for the automated calculation of amplitudes and cross sections in quantum field theory, currently at version 9.3 [11]. We summarize some of the most frequently-used commands here, and refer to [11] for more information

- `Pair[LorentzIndex[`μ`], Momentum[v]]`

  Denotes the four-momentum v^μ. The command `Pair` also allows to input the metric (`Pair[LorentzIndex[`μ`], LorentzIndex[`ν`]]`) and scalar products (`Pair[Momentum[u], Momentum[v]]`).

- `MetricTensor[`μ`, `ν`]`

  Alternative way to input the metric.

- `Eps[LorentzIndex[`μ`],LorentzIndex[`ν`],LorentzIndex[`ρ`], LorentzIndex[`σ`]]`

  Levi-Civita tensor, always assumes upper indices and uses $\epsilon^{0123} = +1$. If contracted with momenta, can write for example `Eps[LorentzIndex[mu], LorentzIndex[nu], Momentum[p], Momentum[q]]` for $\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta$.

- `Contract[`*expr*`]`

  Contracts equal indices in *expr*.

- `DiracGamma[LorentzIndex[mu]], DiracGamma[5], DiracGamma[Momentum[p]]`

  Dirac matrices γ^μ, γ_5, and $\not{p}$.

- `GA[mu], GA[5], GS[p]`

  Like before, but in a shorter (external) notation. `FCI` and `FCE` (for FeynCalcInternal/External) switch between the two notations. Note that the two notations also have different `FullForm`.

- `GA[6], GA[7]`, short-hand notation for the projectors $P_{R/L} = (1 \pm \gamma_5)/2$.

- `GA[mu].GA[nu]` or simply `GA[mu,nu]`. Product $\gamma^\mu \gamma^\nu$ of Dirac matrices

- `DiracSigma[GA[mu, nu]]`, $\sigma^{\mu\nu}$.

- `DiracReduce[`*expr*`]`

  Reduces *expr* to the basis $\{1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}\}$ of bilinear covariants.

- `Tr`[*expr*]

  computes the Dirac trace of *expr*. Warning: Traces without explicit Dirac structures are not evaluated.

- For external Dirac fermions, the syntax for the spinors is
  `SpinorU[p,m]` ,

  `SpinorUBar[p,m]` ,

  `SpinorV[p,m]` ,

  `SpinorVBar[p,m]` .

  Note that the first entry is the momentum and the second is the mass of the particle. The spin index is suppressed.

- For external photons, the syntax for the polarization vectors is
  `PolarizationVector[k,`μ`]` ,

  `Conjugate[PolarizationVector[k,`μ`]]` .

  Again the polarization index is suppressed. To sum over polarizations, use `PolarizationSum[`μ`,`ν`]` .

- For propagators we can use `FeynAmpDenominator[PropagatorDenominator[Momentum[k], m]]` to input a propagator. The short-hand (external) notation would be `FAD[k, m]`. For several propagators, write e.g.

`FeynAmpDenominator[PropagatorDenominator[Momentum[k], m1], PropagatorDenominator[Momentum[k] + Momentum[p], m2]]` or `FAD[k, m1, k + p, m2]`.

- There are various commands to simplify expressions, such as `DiracSimplify`, `DiracReduce`, `Calc`, `DotSimplify`, `DiracOrder`.

The package has also more features such as calculations in the color algebraof $SU(N)$ etc. We omit these features since they are not relevant for this lab course.

7 Tasks for the lab course

The task for the lab course consists of three parts

1. **Derivation of the Feynman rules of QED with the package** `FeynRules`. Create the `FeynRules` model file for QED with two Dirac fermions e and μ, and use the package to derive the corresponding Feynman rules.

2. **Computation of the Born-level differential and total cross section for muon pair creation**. Using the package `FeynArts`, generate the diagrams and the amplitude for the process $e^-(p_1)e^+(p_2) \to \mu^-(p_3)\mu^+(p_4)$ at tree level. Use the picture of the diagrams in your report. Further process the transition amplitude with `FeynCalc` and express your result for the squared and spin-summed amplitude in terms of the Mandelstam variables s, t and u. Finaly, perform the phase-space integration.

3. **Computation of the Born-level differential and total cross section for Compton scattering**. Repeat the steps of 2. for Compton scattering $e^-(p_1)\gamma(p_2) \to e^-(p_3)\gamma(p_4)$. Contrary to the case above, evaluate the differential and total cross section in the lab frame, where the initial electron is at rest. To this end, you can use the formula

$$\frac{d\sigma}{d\Omega_{\text{lab}}} = \frac{1}{64\pi^2\, m_e^2} \left(\frac{\omega'}{\omega}\right)^2 \sum{}' \, |\mathcal{M}|^2 \,,$$

where ω and ω' are the energies of the initial and final state photon in the lab frame, respectively. Moreover, make use of the relation

$$\frac{\omega'}{m_e} = \frac{\frac{\omega}{m_e}}{1 + \frac{\omega}{m_e}(1 - \cos\theta)} \,,$$

which you are supposed to derive from kinematic considerations. θ denotes the angle between the incoming and outgoing photon.

References

[1] S. Weinberg, "The Quantum theory of fields. Vol. 1 and 2," Cambridge University Press, (1995)

[2] M. E. Peskin and D. V. Schroeder, "An Introduction to quantum field theory," Westview Press (1995)

[3] C. Itzykson and J. B. Zuber, "Quantum Field Theory," McGraw-Hill, New York, USA (1980)

[4] J. F. Donoghue, E. Golowich and B. R. Holstein, "Dynamics of the standard model," Cambridge University Press (1992)

[5] M. D. Schwartz, "Quantum Field Theory and the Standard Model," Cambridge University Press (2014)

[6] N. D. Christensen and C. Duhr, Comput. Phys. Commun. **180** (2009), 1614-1641 [arXiv:0806.4194 [hep-ph]].

[7] C. Duhr and B. Fuks, Comput. Phys. Commun. **182** (2011), 2404-2426 [arXiv:1102.4191 [hep-ph]].

[8] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer and T. Reiter, Comput. Phys. Commun. **183** (2012), 1201-1214 [arXiv:1108.2040 [hep-ph]].

[9] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, Comput. Phys. Commun. **185** (2014), 2250-2300 [arXiv:1310.1921 [hep-ph]].

[10] T. Hahn, Comput. Phys. Commun. **140** (2001), 418-431 [arXiv:hep-ph/0012260 [hep-ph]].

[11] V. Shtabovenko, R. Mertig and F. Orellana, Comput. Phys. Commun. **256** (2020), 107478 [arXiv:2001.04407 [hep-ph]].
See also www.feyncalc.org