

Single Photon Source (Photon Statistics)

Objectives:

- In this laboratory experiment students will get familiarized with micro-photoluminescence optical setup and the necessary optical devices for the detection and counting of photons.
- Students will observe the quantum nature of light and understand the underlying principle behind photon number counting and fluctuations.
- Students will verify photon statistics for the demonstration of chaotic light, coherent light and single photon sources.

1. Introduction

Quantum optics study the consequences of considering a beam of light as a stream of photons rather than as a classical wave. It turns out that the differences are rather subtle, and we have to look quite hard to see significant departures from the predictions of the classical theories. In this experiment we will approach the subject from the perspective of the statistical properties of the photon stream. In the following the principle behind photon counting will be presented and further detail can be found in reference [1].

Consider the detection of a light beam by a photon counter as illustrated in Figure 1. The photon counter consists of a very sensitive light detector such as an avalanche photodiode (APD) or photomultiplier tube (PMT) connected to an electronic counter. The detector produces short voltage pulses in response to the light beam and the counter registers the number of pulses that are emitted within a certain time interval set by the user. Photon counters thus operate in a very similar way to the Geiger counters used to count the particles emitted by the decay of radioactive nuclei.

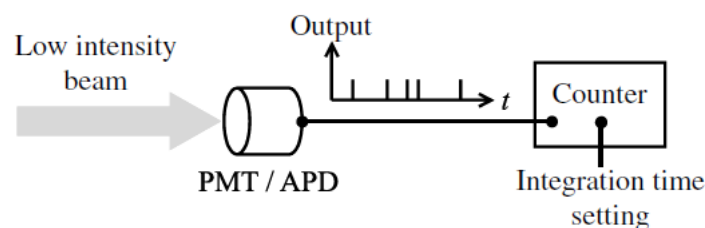


Figure 1. Detection of a faint light beam by a PMT or APD and pulse counting electronics.

The detector emits individual pulses for the impinging light beam that consists of a stream of discrete energy packets that we generally call ‘photons’. The fluctuations in the count rate would give information about the statistical properties of the incoming photon stream.

In this kind of experiments one has to distinguish carefully between:

- (1) the statistical nature of the photo-detection process;
- (2) the intrinsic photon statistics of the light beam.

1.1 Photon-counting statistics

Here we want to answer two fundamental questions 1) Why photon number on short time-scales fluctuates? 2) Is the fluctuation intrinsic or experimental problems?

Let us consider the outcome of a photon-counting experiment as illustrated in Figure. 1. The basic idea is to count the number of photons that strike the detector in a user-specified time interval T . We start with the simplest case and consider the detection of a perfectly coherent monochromatic beam of angular frequency ω and constant intensity I . In the quantum picture of light, the beam to consist of a stream of photons. The photon flux Φ is defined as the average number of photons passing through a cross-section of the beam in unit time. Φ is easily calculated by dividing the energy flux by the energy of the individual photons:

$$\Phi = \frac{IA}{\hbar\omega} \equiv \frac{P}{\hbar\omega} \text{ photons s}^{-1},$$

where A is the area of the beam and P is the power.

Photon-counting detectors are specified by their quantum efficiency η , which is defined as the ratio of the number of photocounts to the number of incident photons. The average number of counts registered by the detector in a counting time T is thus given by:

$$N(T) = \eta\Phi T = \frac{\eta PT}{\hbar\omega}.$$

The corresponding average count rate R is given by:

$$\mathcal{R} = \frac{N}{T} = \eta\Phi = \frac{\eta P}{\hbar\omega} \text{ counts s}^{-1}.$$

The maximum count rate that can be registered with a photon-counting system is usually determined by the fact that the detectors need a certain amount of time to recover after each detection event, which means that a ‘dead time’ of $\sim 70 \text{ ns} - 1 \mu\text{s}$ must typically elapse between successive counts. This sets a practical upper limit on R of around $10^6 \text{ counts s}^{-1}$. With typical values of η for modern detectors of 30-70 % or more, the above equation shows that photon counters are only useful for analysing the properties of very faint light beams with optical powers of $\sim 10^{-12} \text{ W}$ or less. The detection of light beams with higher power levels is done by a different method.

The photon flux and the detector count rate equations given above represent the *average* properties of the beam. A beam of light with a well-defined average photon flux will nevertheless show photon number fluctuations at short time intervals. This is a consequence of the inherent ‘graininess’ of the beam caused by chopping it up into photons. We can see this more clearly with the aid of a simple example. Consider a beam of light of photon energy 2.0 eV with an average power of 1 nW. Such a beam could be obtained by taking a He:Ne laser operating at 633 nm with a power of 1 mW and attenuating it by a factor 10^6 by using neutral density filters filters. The average photon flux is:

$$\Phi = \frac{10^{-9}}{2.0 \times (1.6 \times 10^{-19})} = 3.1 \times 10^9 \text{ photons s}^{-1}.$$

Since the velocity of light is $3 \times 10^8 \text{ m s}^{-1}$, a segment of the beam with a length of $3 \times 10^8 \text{ m}$ would contain 3.1×10^9 photons on average. On a smaller scale, we would expect an average of 31 photons within a 3 m segment of beam. In still smaller segments, the average photon number becomes fractional. For example, a 1-ns count time corresponds to a 30 cm segment of beam, and contains an average of 3.1 photons. Now photons are discrete energy packets, and the actual number of photons has to be an integer. We must therefore have an integer number of photons in each beam segment, as illustrated in Figure 2. In the next section we shall show that if we assume that the photons are equally likely to be at any point within the beam, then we find random fluctuations above and below the mean value. If we were to look at 30 such beam segments, we might therefore find a sequence of photon counts that looks something like:

1, 6, 3, 1, 2, 2, 4, 4, 2, 3, 4, 3, 1, 3, 6, 5, 0, 4, 1, 1, 6, 2, 2, 6, 4, 1, 4, 3, 4, 6. Statistical analysis of this sequence, which is based on uniform random numbers, gives a sum of 95, a mean of 3.16, and a standard deviation of 1.81. The statistical fluctuations arise from the fact that we do not know exactly where the photons are within the beam.

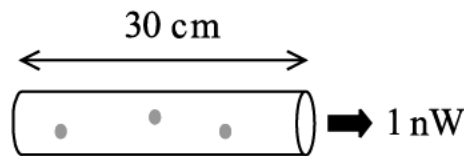


Figure 2. A 30 cm section of a beam light at 633 nm with a power of 1 nW contains three photons on average.

If we make the length of the segments even smaller, the fluctuations become even more apparent. For example, in a 3 cm segment of beam corresponding to a time interval of 100 ps, the average photon number falls to 0.31. The majority of beam segments are now empty, and a sequence of 10 beam segments equivalent to any one of the 30 cm segments considered above might look like: 1, 0, 0, 1, 0, 0, 0, 0, 0, 1. This sequence has a sum of 3, a mean of 0.3, and a standard deviation of 0.46. It is apparent that the shorter the time interval, the more difficult it becomes to know where the photons are. Thus if we split the 30 cm beam segment shown in Figure 2 into 1000 intervals of 0.3 mm length and 1 ps duration, we would find that typically only three intervals contain photons, and 997 are empty. We have no way of predicting which three of these 1000 segments contain the photons. These examples show that although the average photon flux can have a well-defined value, the photon number on short time-scales fluctuates due to the discrete nature of the photons. These fluctuations are described by the photon statistics of the light. In the following sections, we shall investigate the statistical nature of various types of light, starting with the simplest case, namely a perfectly stable monochromatic light source.

1.2 Coherent light: Poissonian photon statistics

In classical physics, light is considered to be an electromagnetic wave. The most stable type of light that we can imagine is a perfectly coherent light beam which has constant angular frequency ω , phase ϕ , and amplitude E_0 :

$$\mathcal{E}(x, t) = \mathcal{E}_0 \sin(kx - \omega t + \phi),$$

where $E(x, t)$ is the electric field of the light wave and $k = \omega/c$ in free space. The beam emitted by an ideal single-mode laser operating well above threshold is a reasonably good approximation to such a field. The intensity I of the beam is proportional to the square of the amplitude, and is constant if E_0 and ϕ are independent of time. There will therefore be no intensity fluctuations and the average photon flux defined earlier will be constant in time.

It might be thought that a beam of light with a time-invariant average photon flux would consist of a stream of photons with regular time intervals between them. This is not in fact the case. We have seen above that there must be statistical fluctuations on short time-scales due to the discrete nature of the photons. We shall now show that perfectly coherent light with a constant intensity has Poissonian photon statistics.

Consider a light beam of constant power P . The average number of photons within a beam segment of length L is given by

$$\bar{n} = \Phi L/c,$$

where Φ is the photon flux. We assume that L is large enough that \bar{n} takes a well-defined integer value. We now subdivide the beam segment into N subsegments of length L/N . N is assumed to be sufficiently large that there is only a very small probability $p = \bar{n}/N$ of finding a photon within any particular subsegment, and a negligibly small probability of finding two or more photons.

We now ask: what is the probability $P(n)$ of finding n photons within a beam of length L containing N subsegments? The answer is given by the probability of finding n subsegments containing one photon and $(N - n)$ containing no photons, in any possible order. This probability is given by the binomial distribution:

$$\mathcal{P}(n) = \frac{N!}{n!(N - n)!} p^n (1 - p)^{N-n},$$

which, with $p = \bar{n}/N$, gives

$$\mathcal{P}(n) = \frac{N!}{n!(N - n)!} \left(\frac{\bar{n}}{N}\right)^n \left(1 - \frac{\bar{n}}{N}\right)^{N-n}.$$

We now take the limit as $N \rightarrow \infty$. To do this, we first rearrange the above equation in the following form:

$$\mathcal{P}(n) = \frac{1}{n!} \left(\frac{N!}{(N-n)!N^n} \right) \bar{n}^n \left(1 - \frac{\bar{n}}{N} \right)^{N-n}.$$

Now by using Stirling's formula:

$$\lim_{N \rightarrow \infty} [\ln N!] = N \ln N - N,$$

we can see that

$$\lim_{N \rightarrow \infty} \left[\ln \left(\frac{N!}{(N-n)!N^n} \right) \right] = 0.$$

Hence:

$$\lim_{N \rightarrow \infty} \left[\frac{N!}{(N-n)!N^n} \right] = 1.$$

Furthermore, by applying the binomial theorem and comparing the result for the limit $N \rightarrow \infty$ to the series expansion of $\exp(-\bar{n})$, we can see that:

$$\begin{aligned} \left(1 - \frac{\bar{n}}{N} \right)^{N-n} &= 1 - (N-n) \frac{\bar{n}}{N} + \frac{1}{2!} (N-n)(N-n-1) \left(\frac{\bar{n}}{N} \right)^2 - \dots \\ &\rightarrow 1 - \bar{n} + \frac{\bar{n}^2}{2!} - \dots \\ &= \exp(-\bar{n}). \end{aligned}$$

On using these two limits, we find

$$\lim_{N \rightarrow \infty} [\mathcal{P}(n)] = \frac{1}{n!} \cdot 1 \cdot \bar{n}^n \cdot \exp(-\bar{n}).$$

We thus conclude that the photon statistics for a coherent light wave with constant intensity are given by:

$$\mathcal{P}(n) = \frac{\bar{n}^n}{n!} e^{-\bar{n}}, \quad n = 0, 1, 2, \dots$$

This equation describes a Poisson distribution. Poissonian statistics generally apply to random processes that can only return integer values.

Poisson distributions are uniquely characterized by their mean value \bar{n} . Representative distributions for $\bar{n} = 0.1, 1, 5, \text{ and } 10$ are shown in Figure 3. It is apparent that the distribution peaks at \bar{n} and gets broader as \bar{n} increases. The fluctuations of a statistical distribution about its mean value are usually quantified in terms of the variance. The variance is equal to the square of the standard deviation Δn and is defined by:

$$\text{Var}(n) \equiv (\Delta n)^2 = \sum_{n=0}^{\infty} (n - \bar{n})^2 \mathcal{P}(n).$$

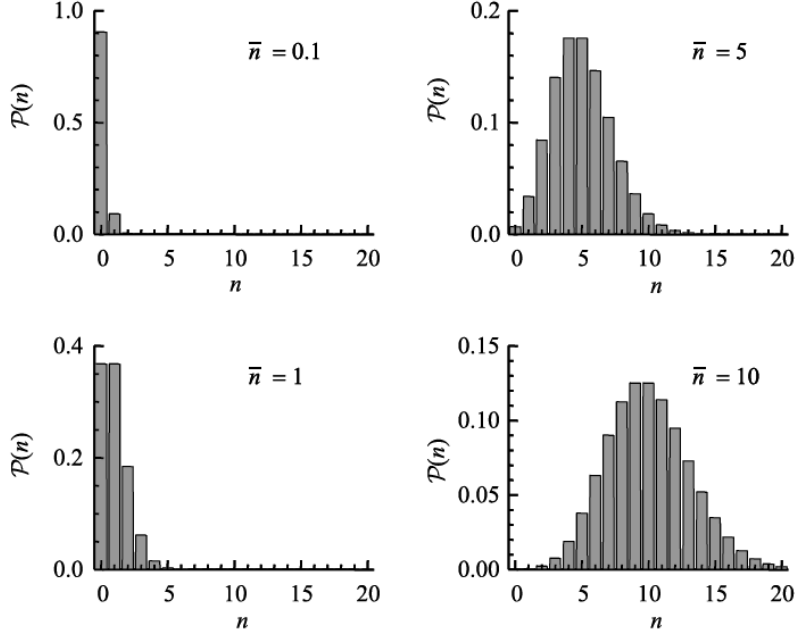


Figure 3 Poisson distributions for mean values of 0.1, 1, 5, and 10. Note that the vertical axis scale changes between each figure.

It is a well-known result for Poisson statistics that the variance is equal to the mean value \bar{n}

$$(\Delta n)^2 = \bar{n}.$$

The standard deviation for the fluctuations of the photon number above and below the mean value is therefore given by:

$$\Delta n = \sqrt{\bar{n}}.$$

This shows that the relative size of the fluctuations decreases as \bar{n} gets larger. If $\bar{n} = 1$, we have $\Delta n = 1$ so that $\Delta n / \bar{n} = 1$. On the other hand, if $\bar{n} = 100$, we have $\Delta n = 10$, and $\Delta n / \bar{n} = 0.1$.

1.3 Classification of light by Photon statistics

In the previous section, we considered the photon statistics of a perfectly coherent light beam with constant optical power P . From a classical perspective, a perfectly coherent beam of constant intensity is the most stable type of light that can be envisaged. This therefore provides a bench mark for classifying other types of light according to the standard deviation of their photon number distributions. In general, there are three possibilities:

- **sub-Poissonian statistics:** $\Delta n < \sqrt{\bar{n}}$,
- **Poissonian statistics:** $\Delta n = \sqrt{\bar{n}}$,
- **super-Poissonian statistics:** $\Delta n > \sqrt{\bar{n}}$.

The difference between the three different types of statistics is illustrated in Figure 4. This figure compares the photon number distributions of super-Poissonian and sub-Poissonian light to that of a Poisson distribution with the same mean photon number. We see that distributions of super-Poissonian and sub-Poissonian light are, respectively, broader or narrower than the Poisson distribution.

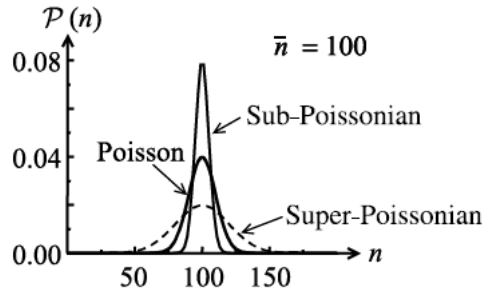


Figure 4. Comparison of the photon statistics for light with a Poisson distribution, and those for sub-Poissonian and super-Poissonian light. The distributions have been drawn with the same mean photon number $\bar{n} = 100$. The discrete nature of the distributions is not apparent in this figure due to the large value of \bar{n} .

It is not difficult to think of types of light which would be expected to have super-Poissonian statistics. If there are any classical fluctuations in the intensity, then we would expect to observe larger photon number fluctuations than for the case with a constant intensity. Since a perfectly stable intensity gives Poissonian statistics, it follows that all classical light beams with time-varying light intensities will have super-Poissonian photon number distributions. Sub-Poissonian light, by contrast, has a narrower distribution than the Poissonian case and is therefore ‘quieter’ than perfectly coherent light.

Now we have already emphasized that a perfectly coherent beam is the most stable form of light that can be envisaged in classical optics. It is therefore apparent that sub-Poissonian light has no classical counterpart, and is therefore the first example of non-classical light that we have met. The table below gives a summary of the classification of light according to the criteria established in this section.

Photon statistics	Classical equivalents	$I(t)$	Δn
Super-Poissonian	Partially coherent (chaotic), incoherent, or thermal light	Time-varying	$> \sqrt{\bar{n}}$
Poissonian	Perfectly coherent light	Constant	$\sqrt{\bar{n}}$
Sub-Poissonian	None (non-classical)	Constant	$< \sqrt{\bar{n}}$

Sub-Poissonian light is defined by the relation:

$$\Delta n < \sqrt{\bar{n}}.$$

It is apparent from Figure 4 that sub-Poissonian light has a narrower photon number distribution than for Poissonian statistics. We have seen that a perfectly coherent beam with constant intensity has Poissonian photon statistics. We thus conclude that sub-Poissonian light is somehow more stable than perfectly coherent light, which has been our paradigm up to this point. In fact, sub-Poissonian light has no classical equivalent. Therefore, the observation of sub-Poissonian statistics is a clear signature of the quantum nature of light. Even though there is no direct classical counterpart of sub-Poissonian light, it is easy to conceive of conditions that would give rise to sub-Poissonian statistics. Let us consider the properties of a beam of light in which the time intervals Δt between the photons are identical, as illustrated schematically in Figure 6. The photocount obtained for such a beam in a time T would be the integer value determined by:

$$N = \text{Int} \left(\eta \frac{T}{\Delta t} \right),$$

which would be exactly the same for every measurement.

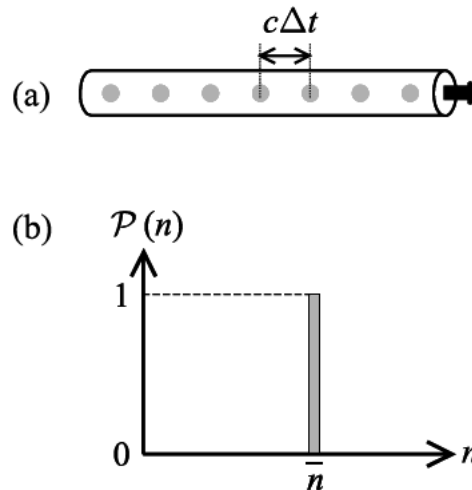


Figure 6. (a) A beam of light containing a stream of photons with a fixed time spacing Δt between them. (b) Photon-counting statistics for such a beam.

The experimenter would therefore obtain the histogram shown in Figure 6(b), with $\bar{n} = N$. This is highly sub-Poissonian, and has $\Delta n = 0$. Photon streams of the type shown in Figure 6(a) with $\Delta n = 0$ are called photon number states. Other types of sub-Poissonian light can be conceived in which the time intervals between the photons in the beam are not exactly the same, but are still more regular than the random time intervals appropriate to a beam with Poissonian statistics. Such types of light are fairly easy to generate in the laboratory, although their detection is quite problematic.

1.4 Degradation of photon statistics by losses

Another important point one should consider during photon statistics is issue related to optical losses.

Let us suppose that we have a beam of light that passes through a lossy medium and is then detected as shown in Figure 7(a). If the transmission of the medium is T , then we can model the losses as a beam splitter with splitting ratio $T : (1 - T)$, as indicated in Figure 7(b). The beam splitter separates the photons into two streams going towards its two output ports, so that only a fraction T of the incoming photons impinge on the detector and are registered as counts. Now the beam splitting process occurs randomly at the level of individual photons, with weighted probabilities for the two output paths of $T : (1 - T)$, respectively. We therefore see that the lossy medium randomly selects photons from the incoming beam with probability T . It is well known that the distribution obtained by random sampling of a given set of data is more random than the original distribution. This point is illustrated in Figure 7(c), which presents the case of a regular stream of photons split with 50 : 50 probability towards two output ports. It is apparent that the regularity of the time intervals in the photon stream going to the detector is reduced compared to the incoming photon stream. Thus the random sampling nature of optical losses degrades the regularity of the photon flux, and would eventually make the time intervals completely random for low values of T .

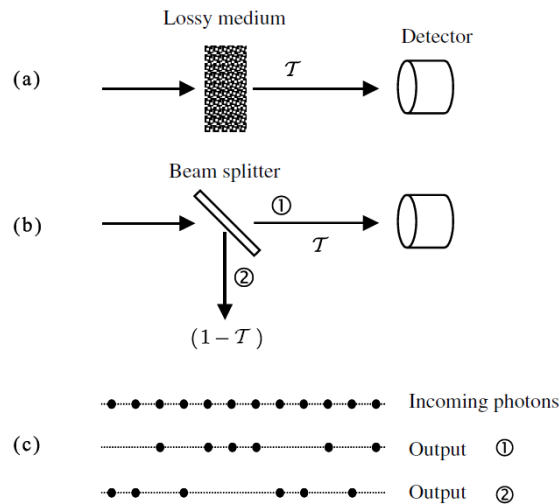


Figure 7 (a) The effect of a lossy medium with transmission T on a beam of light can be modelled as a beam splitter with splitting ratio $T : (1 - T)$ as shown in (b). The beam splitting process is probabilistic at the level of the individual photons, and so the incoming photon stream splits randomly towards the two outputs with a probability set by the transmission: reflection ratio (50: 50 in this case) as shown in part (c).

The beam splitter model of optical losses is a convenient way to consider the many different factors that reduce the efficiency of photon-counting experiments. These factors include:

- (1) inefficient collection optics, whereby only a fraction of the light emitted from the source is collected;
- (2) losses in the optical components due to absorption, scattering, or reflections from the surfaces;
- (3) inefficiency in the detection process due to using detectors with imperfect quantum efficiency

All of these processes are equivalent to random sampling of the photons. The first one randomly selects photons from the source. The second randomly deletes photons from the beam. The third randomly selects a subset of photons to be detected. The first two degrade the photon statistics themselves, while the third degrades the correlation between the photon statistics and the photoelectron statistics. Therefore, we must be very careful to avoid optical losses and use very high-efficiency detectors to observe large quantum effects in the photon statistics.

1.5 Theory of photodetection

Now that we are familiar with the different types of photon statistics that can occur, it is appropriate to consider the relationship between the counting statistics registered by the detector and the underlying photon statistics of the light beam.

Let us consider a photon-counting detector such as an avalanche photodiode or photomultiplier illuminated by a faint light beam. The light interacts with the atoms in the detector and liberates individual electrons by the photoelectric effect. These single photoelectrons then trigger the release of many more electrons in the multiplier region of the tube (as in the case of photomultiplier), thereby generating a current pulse of sufficient magnitude to be detected with an electronic counter.

In this experiment a Single-Photon Avalanche Diode (SPAD) is used for the detection of photons. SPAD defines a class of photodetectors able to detect low intensity signals (down to the single photon) and to signal the time of the photon arrival with high temporal resolution (few tens of picoseconds). SPADs, like the Avalanche photodiode (APD), exploits the photon-triggered avalanche current of a reverse biased p-n junction to detect an incident radiation. The fundamental difference between SPADs and APDs is that SPADs are specifically designed to operate with a reverse bias voltage well above the breakdown voltage (on the contrary APDs operate at a bias lesser than the breakdown voltage). This kind of operation is also called Geiger-mode in literature, for the analogy with the Geiger counter.

SPADs are semiconductor devices based on a p-n junction reversed biased at a voltage higher than the breakdown voltage. At this bias, the electric field is so high that a single charge carrier injected in the depletion layer can trigger a self-sustaining avalanche. The current rises swiftly (sub nanosecond rise-time) to a macroscopic steady level, in the milliamperere range. If the primary carrier is photo-generated, the leading edge of the avalanche pulse marks (with picosecond time jitter) the arrival time of the detected photon. The current continues to flow until the avalanche is quenched by lowering the bias voltage down: the lower electric field is not able any more to accelerate the carriers to impact-ionize with lattice atoms, therefore current ceases. In order to be able to detect another photon, the bias voltage must be raised again above breakdown. These operations require a suitable circuit, which has to i) sense the leading edge of the avalanche current; ii) generate a standard output pulse synchronous with the avalanche build-up; iii) quench the avalanche by lowering the bias down to the breakdown voltage; iv) restore the photodiode to the operative level. This circuit is usually referred to as a quenching circuit.

The SPADs that will be used in this experiment has the following specifications:

Specifications @ 25°C and 5V overvoltage	Min	Typ	Max	Units
Photon Detection Efficiency @ 400nm @ 550nm @ 650nm	21 45 34	24 49 37		%
Single Photon Timing Resolution TTL Counting Output NIM Timing Output (additional internal circuit board required)		35	250 50	ps (FWHM)
After-pulsing probability	0.1		3	%
Dead Time		77		ns
Supply Voltage	unregulated DC, any value 5V ~ 12V			
TTL Output Pulse rise and fall times Output pulse duration Gating input Supply input connector	< 2ns on 10pF load 20ns typical 5V CMOS control (0V, detector off) Standard 3.5mm supply socket			

2. Second order correlation $g^{(2)}(\tau)$ and light sources

In the previous section we classified light beams according to their photon statistics. We saw that the observation of Poissonian and super-Poissonian statistics could be explained by classical wave theory, but not sub-Poissonian statistics. Hence sub-Poissonian statistics is a clear signature of the photon nature of light. In this section we will look at a different way of quantifying light according to the second-order correlation function $g^{(2)}(\tau)$. This will lead to an alternative threefold classification in which the light is described as *antibunched*, *coherent*, or *bunched*. We shall see that antibunched light is only possible in the photon interpretation, and is thus another clear signature of the quantum nature of light. These effects were first investigated in detail by R. Hanbury Brown and R. Q. Twiss (HBT) in the 1950s [2], and their work has subsequently proven to be of central importance in the development of modern quantum optics.

Figure 8(a) illustrates the experimental arrangement for a HBT experiment configured with single photon counting detectors. A stream of photons is incident on a 50 : 50 beam splitter, and is divided equally between the two output ports. The photons impinge on the detectors and the resulting output pulses are fed into an electronic counter/timer. The counter/timer records the time that elapses between the pulses from D1 and D2, while simultaneously counting the number of pulses at each input. The results of the experiment are typically presented as a histogram, as shown in Figure 8(b). The histogram displays the number of events that are registered at each value of the time τ between the start and stop pulses.

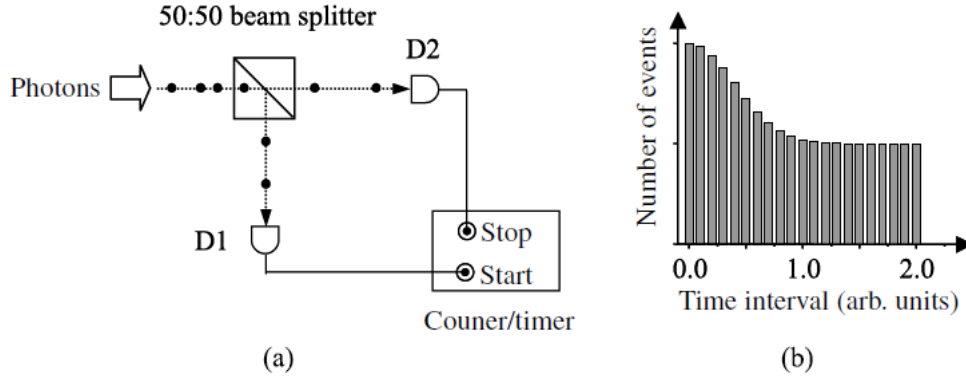


Figure 8. (a) Hanbury Brown–Twiss (HBT) experiment with a photon stream incident on the beam splitter. The pulses from the single-photon counting detectors D1 and D2 are fed into the start and stop inputs of an electronic counter/timer. The counter/timer both counts the number of pulses from each detector and also records the time that elapses between the pulses at the start and stop inputs. (b) Typical results of such an experiment. The results are presented as a histogram showing the number of events recorded within a particular time interval. In this case the histogram shows the results that would be obtained for a bunched photon stream.

Since the number of counts registered on a photoncounting detector is proportional to the intensity, one can write the second order correlation function of $g^{(2)}(\tau)$ as:

$$g^{(2)}(\tau) = \frac{\langle n_1(t)n_2(t + \tau) \rangle}{\langle n_1(t) \rangle \langle n_2(t + \tau) \rangle},$$

where $n_i(t)$ is the number of counts registered on detector i at time t . This shows that $g^{(2)}(\tau)$ is dependent on the simultaneous probability of counting photons at time t on D1 and at time $t + \tau$ on D2. In other words, $g^{(2)}(\tau)$ is proportional to the conditional probability of detecting a second photon at time $t = \tau$, given that we detected one at $t = 0$. This is exactly what the histogram from the HBT experiment with photoncounting detectors records. Hence the results of the HBT experiment also give a direct measure of the second-order correlation function $g^{(2)}(\tau)$ in the photon interpretation of light.

A moment's thought makes us realize that completely different results are possible with photons at the input port of the beam splitter than with a classical electromagnetic wave. Let us suppose that the incoming light consists of a stream of photons with long time intervals between successive photons. The photons then impinge on the beam splitter one by one and are randomly directed to either D1 or D2 with equal probability. There is therefore a 50% probability that a given photon will be detected by D1 and trigger the timer to start recording. The generation of a start pulse in D1 implies that there is a zero probability of obtaining a stop pulse from D2 from this photon. Hence the timer will record no events at $\tau = 0$. Now consider the next photon that impinges on the beam splitter. This will go to D2 with probability 50%, and if it does so, it will stop the timer and record an event. If the photon goes to D1, then nothing happens and we have to wait again until the next photon arrives to get a chance of having a stop pulse. The process proceeds until a stop pulse is eventually achieved. This might happen with the first or second or any subsequent photon, but never at $\tau = 0$. We therefore have a situation where we

expect no events at $\tau=0$, but some events for larger values of τ , which clearly contravenes the classical results. We thus immediately see that the experiment with photons can give results that are not possible in the classical theory of light. The observation of the non-classical result with $g^{(2)}(\tau) = 0$ arose from the fact that the photon stream consisted of individual photons with long time intervals between them. Let us now consider a different scenario in which the photons arrive in bunches. Half of the photons are split towards D1 and the other half towards D2. These two subdivided bunches strike the detectors at the same time and there will be a high probability that both detectors register simultaneously. There will therefore be a large number of events near $\tau=0$. On the other hand, as τ increases, the probability for getting a stop pulse after a start pulse has been registered decreases, and so the number of events recorded drops. We thus have a situation with many events near $\tau=0$ and fewer at later times, which is fully compatible with the classical results.

In general there are threefold classification of light according to the second-order correlation function $g^{(2)}(\tau)$. The classification is based on the value of $g^{(2)}(0)$ and proceeds as follows:

- **bunched light:** $g^{(2)}(0) > 1$,
- **coherent light:** $g^{(2)}(0) = 1$,
- **antibunched light:** $g^{(2)}(0) < 1$.

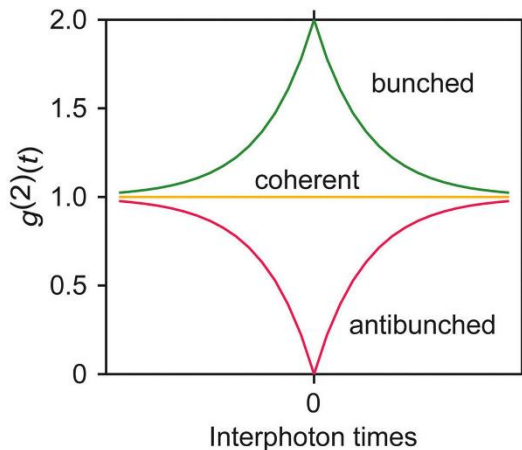


Figure 9. The second-order coherence function $g^{(2)}$ versus interphoton time for antibunched, coherent (laser), and bunched (thermal) light sources. $g^{(2)}(0)=0$ means that all the photons are separated in time.

This point is summarized in Figure 9 and in the following table.

Classical description	Photon stream	$g^{(2)}(0)$
Chaotic	Bunched	>1
Coherent	Random	1
None	Antibunched	<1

Figure 10 is a simplistic attempt to illustrate the difference between the three different types of light in terms of the photons streams. The reference point is the case where the time intervals between the photons are random. On either side of this we have the case where the photons

spread out with regular time intervals between them, or where they clump together in bunches. These three cases correspond to coherent, antibunched, and bunched light, respectively. In what follows below, we explore the properties of each of these three types of light in more detail, starting with coherent light.

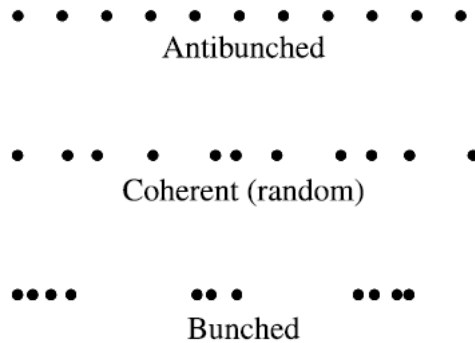


Figure 10. Comparison of the photon streams for antibunched light, coherent light, and bunched light. For the case of coherent light, the Poissonian photon statistics correspond to random time intervals between the photons.

2.1 Single Photon Source and photon antibunching

We have seen above that the observation of photon antibunching is a clear proof of the quantum nature of light. The basic principle of an antibunching experiment is to isolate an *individual* emitting species (i.e. an individual atom, molecule, quantum dot, or colour centre) and regulate the rate at which the photons are emitted by fluorescence. This is done by shining a laser onto the emissive species to excite it, and then waiting for the photon to be emitted. Once a photon has been emitted, it will take a time approximately equal to the radiative lifetime of the transition, namely τ_R , before the next photon can be emitted. This leaves long time gaps between the photons, and so we have antibunched light. We can understand this process in more detail by referring to Figure 11, which shows a schematic representation of the photon emission sequence from a single atom. Let us suppose that the atom is promoted to an excited state at time $t = 0$, as indicated by the dashed line. The emission probability of the transition dictates that the average time to emit the photon is equal to τ_R . Once the photon has been emitted, the atom can be re-excited by the laser, which will only require a short amount of time if a high-power laser is used. The atom can then emit another photon after a time $\sim \tau_R$, at which point the excitation–emission cycle can start again. Since spontaneous emission is a probabilistic process, the emission time will not be the same for each cycle, which means that the stream of photons will not be exactly regular. However, it is clear that the probability for the emission of two photons with a time separation τ_R will be very small. There will therefore be very few events when both the start and stop detectors of the HBT correlator in Figure 11 fire simultaneously, and so we shall have $g^{(2)}(0) \approx 0$.

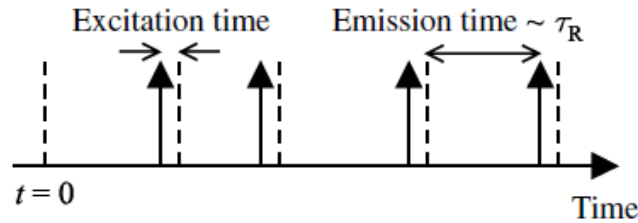


Figure 11. Schematic representation of the photon emission sequence from a single atom excited by an intense laser. The dashed lines indicate the times at which the atom is promoted to the excited state, while the arrows indicate the photon emission events. τ_R is the radiative lifetime of the excited state.

Antibunching has also been observed from many other types of light emitters, including a number of solid-state sources, such as:

- fluorescent dye molecules doped in a glass or crystal;
- semiconductor quantum dots;
- colour centres in diamonds.

An application of the techniques for the generation of antibunched light is the development of a triggered single-photon source. These sources are needed to improve the security in quantum cryptography experiments. The basic idea of a single-photon source is that the source should emit exactly one photon in response to a trigger pulse, which can be either electrical or optical. The operating principle is shown in Figure 12. The source consists of a single emissive species (say an atom), and the trigger pulse excites the atom to an upper excited state, as shown in Figure 12(a). The atom then emits a cascade of photons as it relaxes to the ground state. Since the photons have different wavelengths, it is possible to select the photon from a particular transition by filtering the fluorescence. There will only ever be one photon emitted from a specific transition in each cascade.

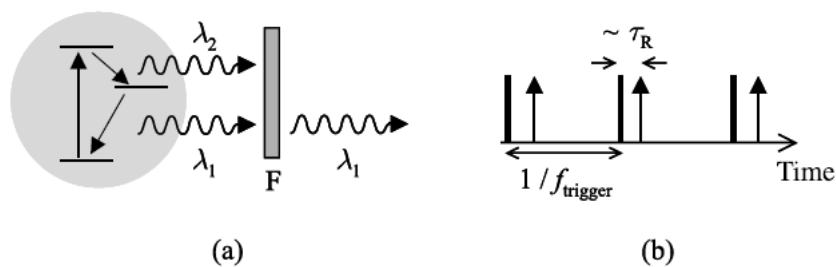


Figure 12. Excitation–emission cycle from a single atom in response to trigger pulses. (a) The atom emits a cascade of photons of different wavelengths as it relaxes, but by using a suitable filter (F), only one of them is selected. (b) Schematic representation of the photon emission sequence. Trigger pulses are indicated by the thick lines, while the arrows indicate the photon emission events, which occur at a time roughly equal to the radiative lifetime (τ_R) after the trigger pulses. When the pulse separation $1/f_{\text{trigger}}$ is significantly longer than τ_R , the photon stream is controlled by the trigger pulse sequence.

Consider now the timing of the photons emitted by this process. An intense trigger pulse will rapidly promote an electron to the excited state, and the atom will emit exactly one photon after a time roughly equal to the radiative lifetime τ_R , as shown schematically in Figure 12(b). No more photons can be emitted until the next trigger pulse arrives, when the process repeats itself. The time separation of the trigger pulses is determined by the frequency f_{trigger} at which the trigger source operates. If the time separation between the pulses is significantly longer than τ_R , the trigger pulses control the separation of the photons in the fluorescence. We thus have a source that emits exactly one photon of a particular wavelength whenever a trigger pulse is applied. The easiest way to make a triggered single-photon source is to use an optical trigger from a suitable laser. However, in the long run it will be important to develop electrically triggered devices.

3. *Experimental background and Procedures*

Figure 13 depicts schematic of the experimental setup used to measure the photon statistics of different light sources.

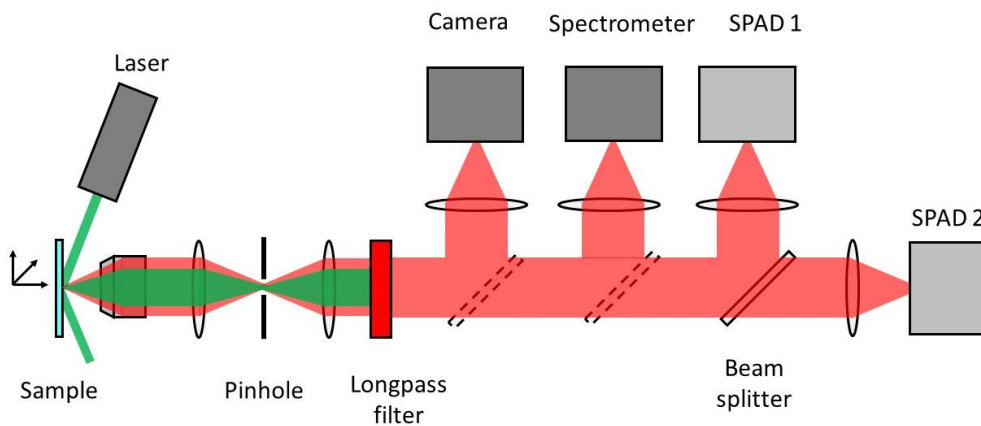


Figure 13. Schematic of the optical setup for measuring the photon statistics of different light sources.

The quantum emitters will be excited by a 532 nm cw laser at a power $< 1\text{mW}$ (CPS 532 - Collimated Laser Diode Module, Thorlabs GmbH) at an angle as shown in Figure 13 (green). The laser is filtered by a laser line filter that has a FWHM of 3 nm (FL532-3, Thorlabs GmbH) to have a clean excitation. A neutral density filter with variable optical density controls the excitation intensity. The fluorescence photons from the sample (red) is at a higher wavelength and is collected via a microscope objective (Zeiss Objective C Epiplan-Apochromat 20X/0.7 DIC). The collected signal is spectrally filtered using two long pass filters that have cut-on wavelength at 550 nm (FELH0550, Thorlabs GmbH) to suppress the scattered laser light. A confocal pinhole of 50 μm diameter selects the focal plane of the emitter and gets rid of the unwanted background signal. A flippable reflecting mirror that is placed along the optical axis of the setup sends the fluorescence signal to the imaging camera. Imaging helps to identify the quantum emitters and align them along the optical axis for detection. The spectrometer (Flame-S-VIS-NIR-ES, Ocean Optics GmbH) is used to investigate the spectral feature of the emitters. A 50/50 non-polarizing beam splitter sends the emitted photons to the two single-photon

avalanche photodiodes (SPADs) (Micro Photon Devices, 100 μm active area, < 50 ps jitter and < 250 cps dark count). These detectors are connected to the start-stop time-interval analyzer of a time-correlated single photon counter (TCSPC) (TimeHarp 260 Nano, PicoQuant GmbH) and the delay between the arrival times of two emitted photons will be repeatedly measured and histogrammed with picosecond time resolution to determine the photon statistics. Figure 14 shows the real experimental setup.

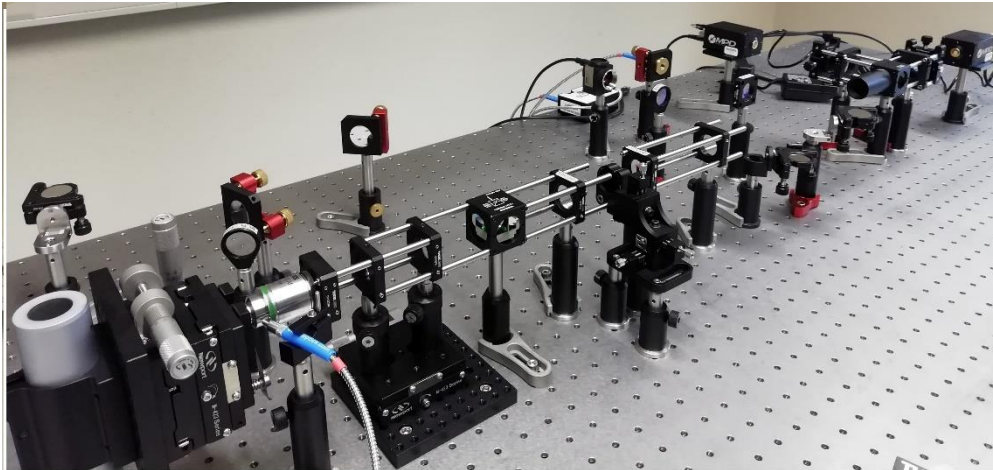


Figure 14. Experimental setup for measuring the second order correlation $g^{(2)}(\tau)$ of different light sources.

Note that the technical note [3] gives a theoretical overview about Time-Correlated Single Photon Counting (TCSPC). It provides information about basic principles such as count rate statistics, pile-up limitation or reverse start-stop mode. The overall timing resolution (Instrument Response Function - IRF) as well as how individual components of a TCSPC setup contribute to it is also explained. The general characteristics of photon counting detectors are also discussed.

Procedures and students' activity:

1. Check the necessary optical devices mentioned above and determine the polarization of the laser and its power.
2. Using the neutral density filter to decrease the laser power and excite the fluorescent sample. Use the camera to identify the emitters and align precisely along the optical axis for detection.
3. Measure the fluorescence spectrum using the spectrometer.
4. Determine the dark count of the detectors and optimize the setup to achieve the specified value (the specification sheet is on the table).
5. Use the HBT interferometer configured with single photon counting detectors to record the number of events that are registered at each value of the time τ between the start and stop pulses.
6. Chaotic, coherent and single-photon light sources will be provided and determine the photon statistics and measure the second order correlation function.

7. Explain why the second order correlation function measurement for a single-photon source deviate from the ideal $g^{(2)}(0) = 0$ result.

4. Data Analysis and Report format

A good lab report does more than presenting data; it demonstrates the writer's comprehension of the concepts behind the data. Merely recording the expected and observed results is not sufficient; you should also identify how and why differences occurred, explain how they affected your experiment, and show your understanding of the principles the experiment was designed to examine. Bear in mind that a format, however helpful, cannot replace clear thinking and organized writing. You still need to organize your ideas carefully and express them coherently.

Typical components to be fulfilled are: Title Page, Abstract, Introduction, Methods and Materials (or Equipment), Experimental Procedure, Results, Discussion, Conclusion, References.

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References

1. M. Fox, Quantum Optics: An Introduction, Oxford Master Series in Physics (2006).
2. R. Hanbury Brown, R. Q. Twiss, A Test of a New Type of Stellar Interferometer on Sirius, Nature, 178, 1046 (1956).
3. M. Wahl, Time-Correlated Single Photon Counting, PicoQuant Technical Note (2014) (http://www.picoquant.com/images/uploads/page/files/7253/technote_tcspc.pdf).