Nuclear Lifetime: Measurement of the Lifetime of the Exited 14.4 keV Level of ^{57}Fe

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1 Introduction

The lifetimes of nuclei depend on the type of interaction (strong, weak or electromagnetic) responsible for the transition. In addition, the change of energy, angular momentum and parity strongly influence the transition rates. Using models for the nuclear structure and the interaction, it is possible to calculate these transition rates. The measurements thus allow to test the theoratical assumption and to further develop the underlying models.

In the setup used in our experiment we use the method of delayed coincidence to measure the lifetime of the excited 14.4 keV level of the nucleus ^{57}Fe. The transition is accompanied by a γ ray emission and therefore is of electromagnetic nature.

2 Theoretical Considerations

2.1 Gamma Radiation

Whereas light and X rays are produced by de-excitation of atomic electron states, gamma rays are produced by transitions from excited nuclear states (energies in the keV range up to several MeV) and by decays or interactions of elementary particles (energies in the MeV range). Photons of energies up to 10^{20} are observed in cosmic ray experiments

In our experiment the gamma rays are. generated by de-excitation of excited ^{57}Fe nuclei which are produced through electron capture (ec) of ^{57}Co (Fig: 2.1).

The energy difference between the two states involved in the transition is transferred to the photon except for a small fraction taken by the recoiling nucleus. The nuclear states being quite narrow, the photon energy takes well defined values, we observe energy lines. Often the de-excitation occurs in several steps passing through intermediate states.

The decay scheme of Figure 2.1 shows the transitions from the 136 keV excited state to the ground state. In addition to the 136 keV photons also 122 keV and 14.4 keV photons are produced. Figure 2.2 shows a measured photon energy spectrum.

Due to the finite resolution the two high energy lines 136 keV and 122 keV are not separated. In addition to the lines predicted by our decay scheme we expect the Iodine escape peak (the detector is a NaI crystal), a Compton

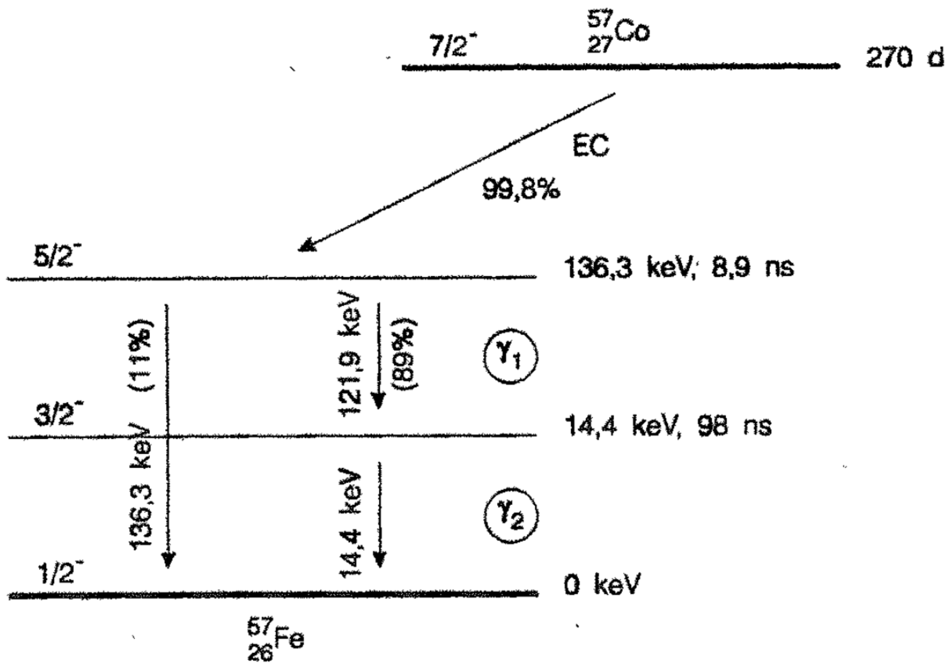

Figure 2.1: Decay scheme of ^{57Co}.

cusp and at low energies X-ray fluorescent lines of ^{57}Fe (K_α, K_β, ...) at energies of 6.4 keV and 7.06 keV. The resolution of the detector is not sufficient to identify these details.

2.2 Decay Distribution

Like all radioactive decays and decays of instable particles the de-excitation is a spontaneous, purely statistical process. It is not possible to predict the time when a specific nucleus decays, we can only state the probability dP for a decay in a certain time interval. It is proportional to the length of the interval dt and is by definition equal to the expected fraction dN/N of the decaying nuclei, where $N(t)$ is the number of existing nuclei at time t.

$$dP = -\frac{dN}{N} = \lambda t \qquad (2.1)$$

The minus sign expresses the that the is decreasing. The proportionality

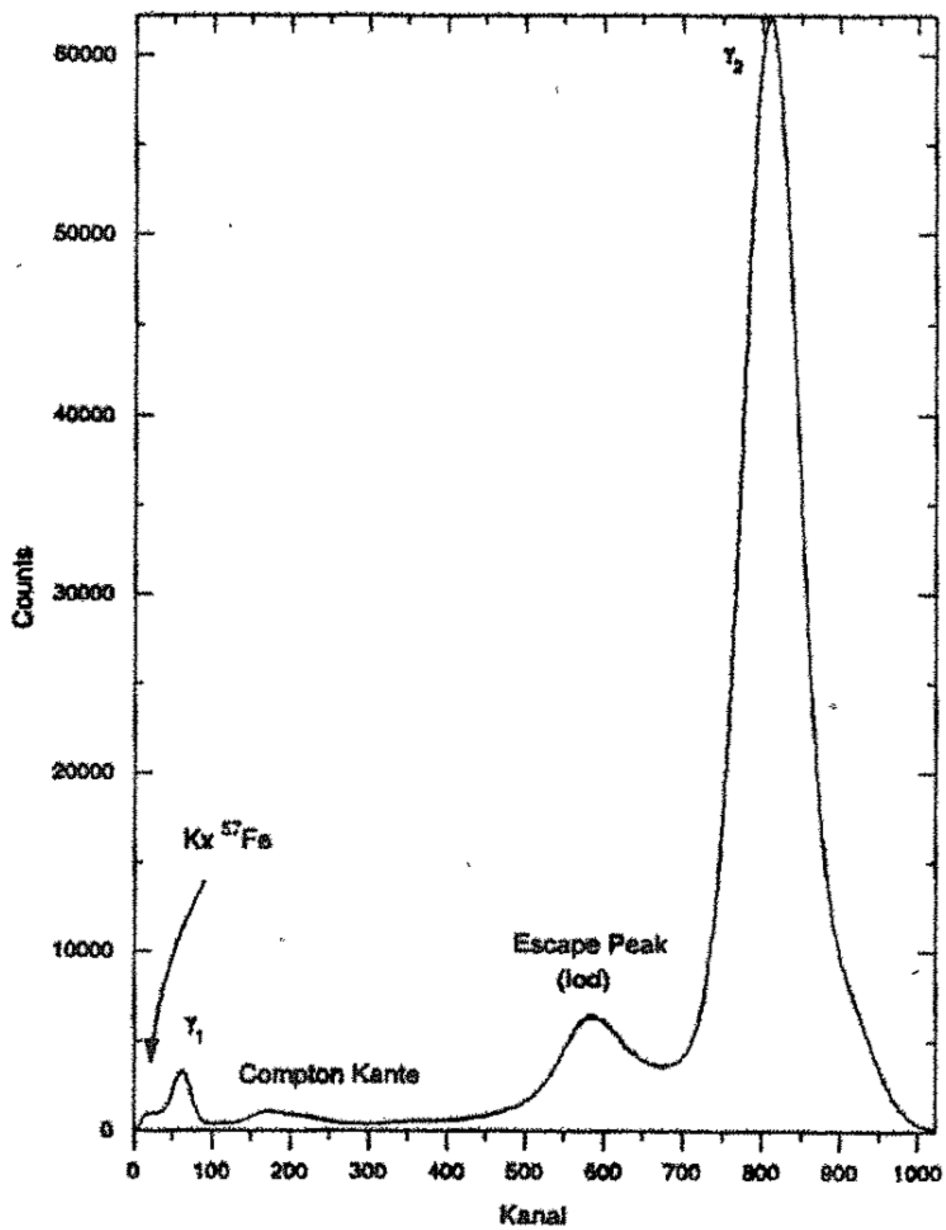

Figure 2.2: The γ spectrum of ^{57Co}.

factor λ is called decay constant. High decay probabilities correspond to large decay constants.

We can compute the decay constant if we know the interaction operator H using Fermi's golden rule:

$$\begin{aligned} \lambda &= \int_0^{E_{max}} W_{fi} dE \\ W_{fi} &= \frac{2\pi}{h}\rho(E)|\langle\psi_f \mid H \mid \psi_i\rangle|^2 \end{aligned} \tag{2.2}$$

Here are E the energy of the final état photon, W_{fi} the transition rate from the initial state i to the final state f and $\psi_f,\ \psi_i$ the states of the nucleus after and before the transition.

The solution of Equation 2.1 is

$$N(t) = N_0 e^{-\lambda t} \tag{2.3}$$

where N_0 is the number of nuclei at time zero. The fraction N/N_0 is displayed in Fig. 2.3. From Equation 2.1 we derive the normalized decay rate $f(t) = dP/dt$.

$$f(t) = \lambda e^{-\lambda t} \tag{2.4}$$

This funtion is the probability density for the decay time. The total probability for a decay at any time is obviously one:

$$\int_0^{\infty} f(t)dt = \int_0^{\infty} \lambda e^{-\lambda t} dt = 1 \tag{2.5}$$

The mean lifetime τ is the expression value of the decay time:

$$\begin{aligned} \tau &= \int_0^{\infty} t\ f(t)dt \\ &= \int_0^{\infty} t\lambda e^{-\lambda t} dt = \frac{1}{\lambda} \end{aligned} \tag{2.6}$$

It is the universe of the decay rate. In the literature we also find the half-life t_h of nuclei. It corresponds to the time where half of the initially existing nuclei have decayed.

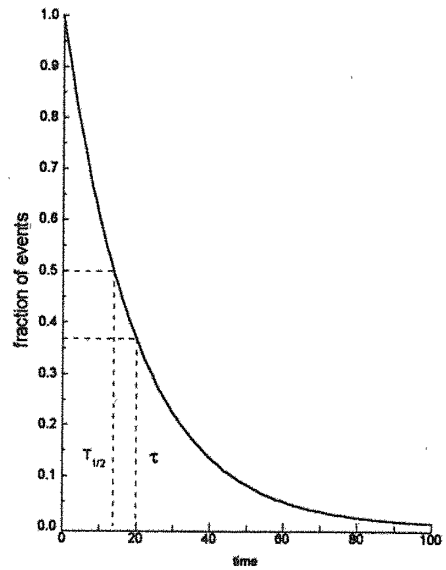

Figure 2.3: Exponential decay distribution.

3 Experimental Method

3.1 Principle

A large variety of methods is used to measure lifetimes of instable nuclei or other particles with widely varying lifetimes. When the lifetimes are in the range of nanoseconds or milliseconds then we can apply the method of delayed coincidence. A clock is started by a process which is related to the production of the decaying object and stopped by the registered decay. In our case the start signal is produced by the 121.9 keV photon produced together with the excited 14.4 keV Fe nucleus and the stop signal is derived from the 14.4 keV photon emitted at the de-excitation.

When we collect the decays in a histogram of decay times with fixed bin width, the histogram follows the exponential decay rate distribution. In a logarithmic representation the measured event numbers follow a straight line as indicated in Fig. 3.1. and the slope is just the decay rate λ

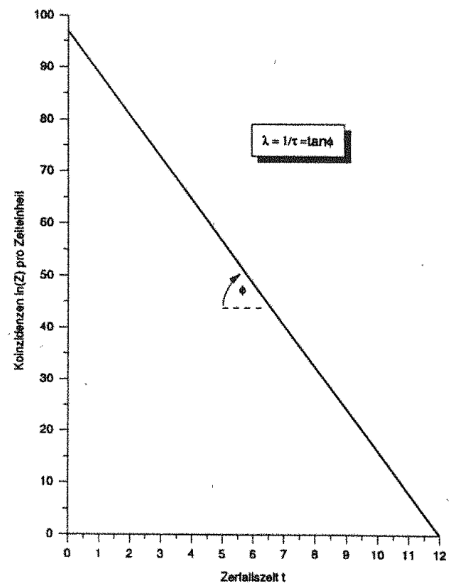

Figure 3.1: Decay time distribution in logarithmic representation.

3.2 Experimental Setup

The experimental setup is shown schematically in Fig. 3.2. We use scintillators (Sz) to detect the photons. Atoms of the scintillators are excited by ionizing particles. Transitions from the excited state through intermediate levels to the ground state produce photons in the visible or UV spectral range. The ionization is caused by electrons liberated from the atoms by the photons by photo-effect or Compton scattering. The amount of emitted light is proportional to the energy deposition produced by the photon in the scintillator.

The scintillation light is transformed to an electric signal by photomultipliers (PM). The photons hit the photocathode and produce electrons. The efficiency is about 10% and depends strongly on their wave length. The primary electrons are sucked by an electric field and accelerated towards the first dynode. There each electron produces an average of two to three secondary electrons which are in turn accelerated to the second dynode a.s.o. The final result from the cascade of 10 to 14 dynodes is a charge amplification by a factor of 10^5 to 10^9 at the anode. The anode current is further amplified by a charge sensitive pre-amplifier.

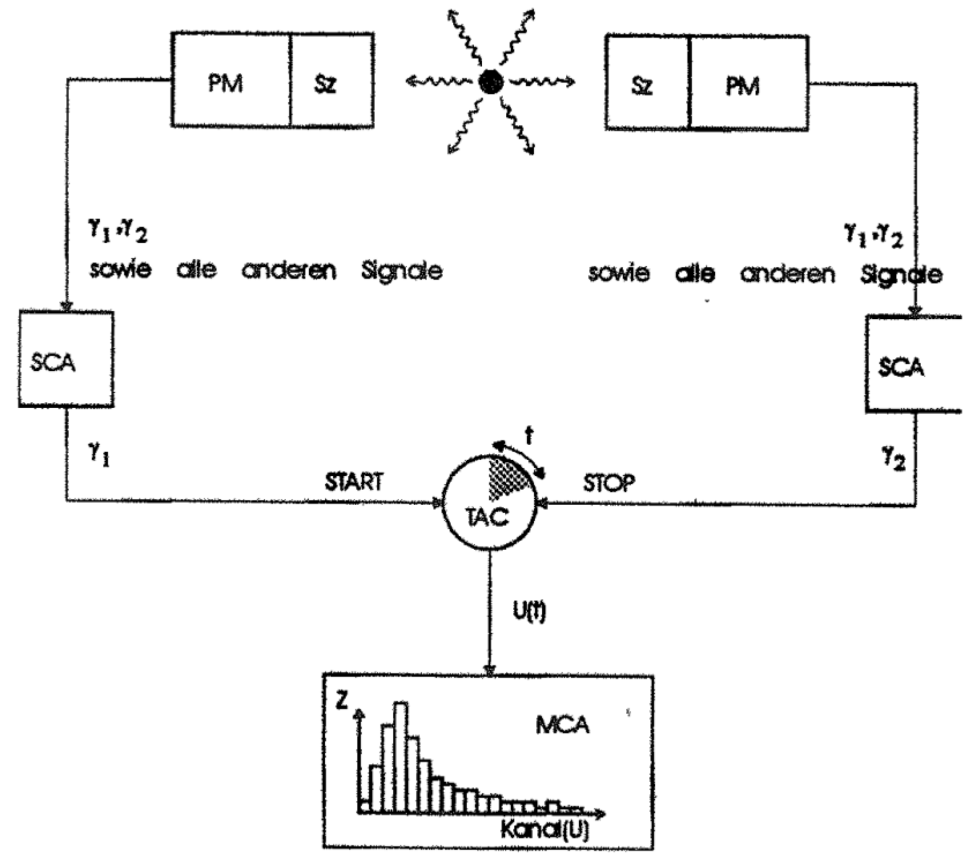

Figure 3.2: The principle of the measurement.

The three components, scintillator, photomultiplier and pre-amplifier are housed in one unit to avoid as much as possible pick-up and electronic noise. This is achieved by short signal cables and a good shielding. The signal from the pre-amplifier is shaped by the main amplifier (HV). The pulse shaping allows deduce from the signal the arrival time and the energy of the photon. When the pulse height lies in the energy window of the (adjustable) single channel analyzer (SCA) the latter produces a square pulse used to start or stop the time to amplitude converter (TAC) (see Fig. 3.3).

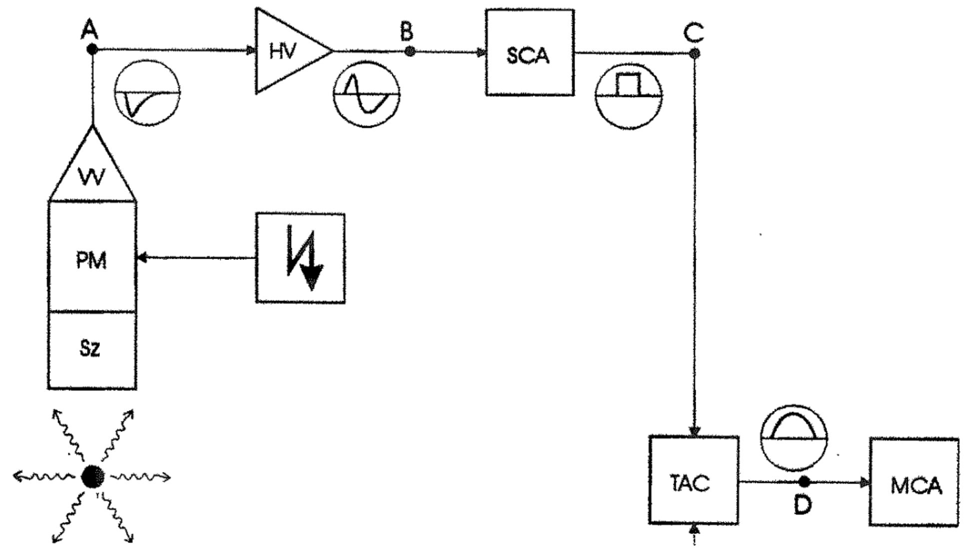

Figure 3.3: Block diagram of electronic units of one branch of the setup.

3.3 Checking the Circuit and the Units

To check the performance of the electronic setup we start with some test using the oscilloscope and multi channel analyzer (MCA). The tests have to be performed for each of the units in both branches of the setup.

First we switch on the voltage of the crate which delivers the low voltage for the NIM units:

Attention:

- Never switch on the crate without the assistantat physicist being present.
- Before switching off the crate, the high voltage has to be down to zero and the HV supplies have to be switched off to avoid overvoltages in the PMs.
- Do not insert or take out modules while the crate power is on.

After connection the connector unit, we switch on the HV of the PMs.

Attention:

- Before switching off the HV put the potentiometer to zero.

Now we display the output signal of the pre-amplifier at the scope connecting it to point A (see Fig. 3.3). Increasing the high voltage we observe a signal similar to the one shown in Fig. 3.4 as long as the amplifier is not saturated.

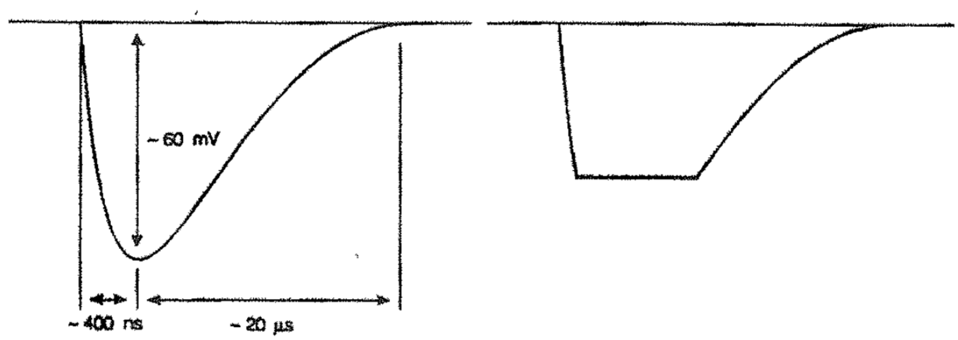

Figure 3.4: Output signals from the pre-amplifier: a)correct, b) saturated.

Attention:

- The HV must not exceed 1 kV (relevant is the potentiometer reading, not the display of the meter).

The slope of the rising signal contains the physical information: The amplitude corresponds to the charge collected in the PM (and thus also to the intensity of the scintillation light and therefore also to the photon energy). For a constant rise time the total charge is represented by the slope. The slope is used to produce fast signals by the main amplifier and shaper.

Advice:

- Select the HV of the PM as high as possible to improve the signal to noise ratio and the time resolution.

Now we connect the main amplifier to the pre-amplifier and measure the signal at point B. The main amplifier's task is to shape the signal to provide the short pulses independent from the amplitude which are needed to digest the high rates.

These tasks are solved by differentiating the pre-amplifier signals. In this way we obtain signals as shown in Fig. 3.5 which vary only in the amplitude and provide a measure of the charge and thus for the γ energy. The crossing time of the axis is almost independent of the signal height and the rise time is shorter than that of the pre-amplifier.

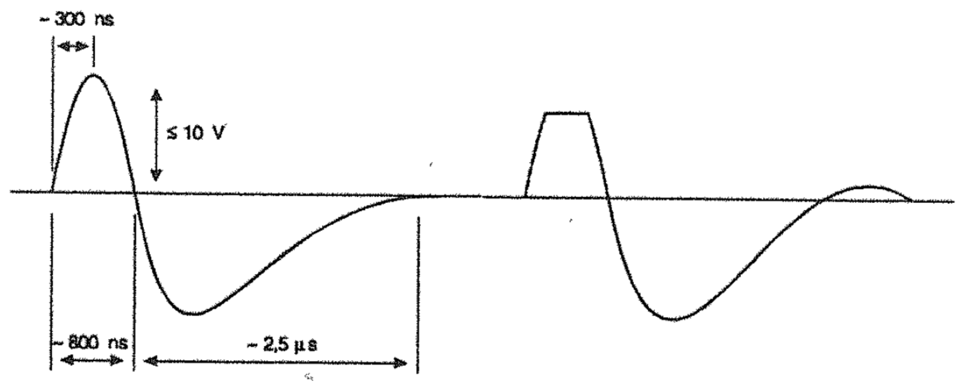

Figure 3.5:

The input range of the MCA input is below 10 V (see below). Therefore the gain of the main amplifier is chosen in such a way as to guarantee pulse heights not exceeding 10V. You have to avoid a saturation of the amplifier as indicated in Fig. 3.5.

In order to register the γ spectra the oscilloscope is replaced by the MCA. The input range of our MCA is from zero to 10 V. The input signals are histogrammed into the 1023 channels according to the pulse height. The histogram is presented graphically at the screen. It can be read-out by a computer. The first channel (channel 0) displays the time (in seconds) used to collect the data.

Attention:

- Do not move the PM during the measurement. The PM is not shielded with μ metal against the magnetic field. Moving the PM can affect the influence of the magnetic field on the amplification.

3.4 Signal Discrimination

Each of the two electronic branches has to select one of the two γ lines. We use the circuit of Fig. 3.6.

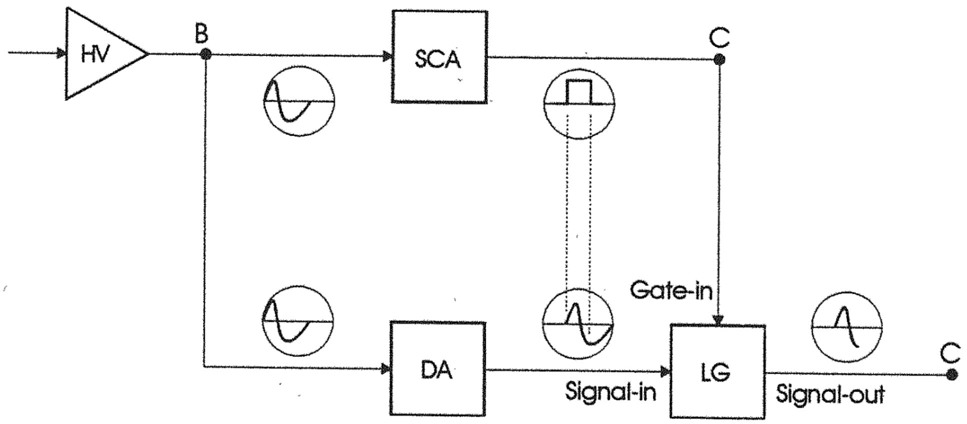

Figure 3.6: Circuit used to detect γ peaks.

At point B (see Fig. 3.3) the output signal is divided. One branch contains only a single channel analyzer (SCA). Its working principle is explained in Fig. 3.7.

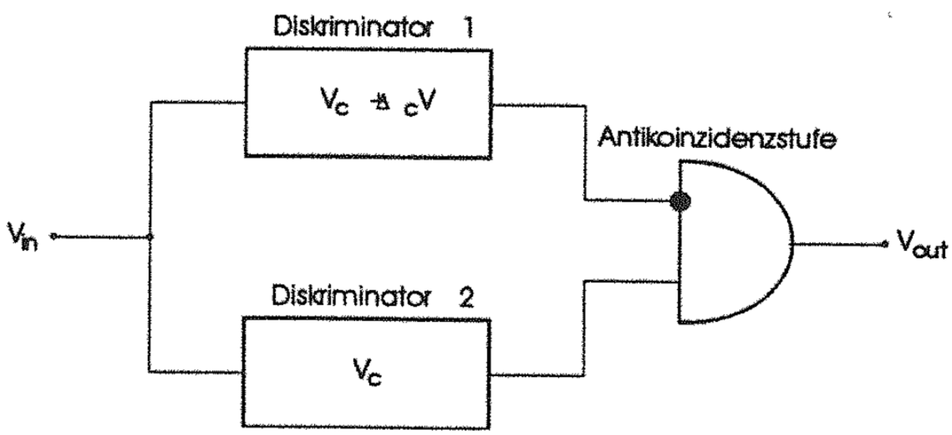

Figure 3.7: Working principle of single channel discriminator.

The input signal of the SCA is fed into two parallel discriminators. One of the discriminators responds above a threshold of V_c, the other one at a threshold of $V_c + \Delta V_c$. The logical output (NIM or TTL convention) of the second discriminator is inverted and put in coincidence with the logic output of the first one. Thus a "true" signal is produced only if the input signal V_{in} fulfills $V_c < \Delta V_{in} < V_c + \Delta V_c$.

The SCA provides two outputs: a slow output (slow-out, TTL) of rectangular and a fast output (fast-out, NIM) at a fixed delay after the zero crossing of the input signal.

The lower branch in Fig. 3.6 consists of a delay amplifier (DA) and a linear gate (LG). The linear gate is opened by a trigger pulse fed into the gate (gate in). During the time to interval of adjustable length started by the gate pulse the input is connected to the output of(see Fig. 3.8). The principle of operation is similar to that of a switched transistor. Also the start of the time window can be delayed (gate delay).

The gate input of the LG is connected to the output of the SCA (slow out) and the signal input of the LG to the output of the DA which provides the signal from the main amplifier. This signal passes if the gate is open which means that the signal has a signal height in the range selected by the SCA. The delay of the DA and the width of the gate window of the linear gate are adjusted such that only the positive part of the input signal appears at the output.

The upper and lower limits of the SCA are selected such that only the selected γ signal passes. The energy selection is either done with the help of the oscilloscope or the MCA. First we open the SCA completely and observe the full γ spectrum at the MCA screen. By rising the lower threshold and lowering the upper threshold of the SCA we can eliminate all signals except those corresponding to the γ_1 line (or γ_2 line for the second SCA).

For the following measurement we need only the time information and use only the fast output of the SCA (Fig. 3.3). The lower branch of the circuit of Fig. 3.6 has only the purpose to control the correct adjustment of the SCA thresholds. We replace it now by a resistor. The resistor is required, since the presence of the lower branch reduces the output signal of the amplifier. Removing it without replacement would change the energy interval selected by the SCA.

The required resistivity is adjusted in the following way (Fig. 3.9): We display a line of the spectrum at the MCA with the DA in. The cursor is moved on top of the peak. Then we replace the DA by the resistor and we will realize that the peak migrates to one side. We adjust the resistivity until the peak is again at the cursor position.

This procedure has to be followed for both energy branches.

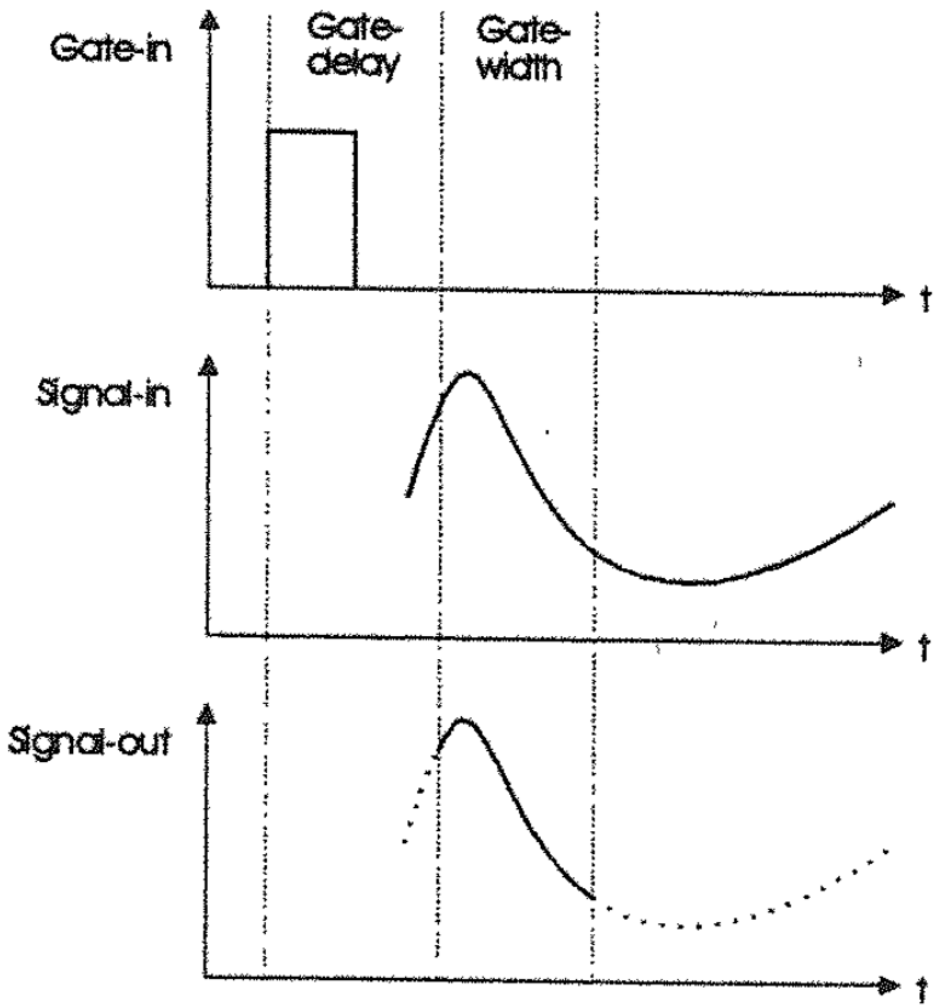

Figure 3.8: Signal diagram of a linear gate.

4 Experimental Procedure

To measure the lifetime we connect the fast output of one SCA (γ_1 signal) to the start input of the time to amplitude converter (TAC) and the other one (γ_2 signal) to the stop input.

The TAC contains a capacitor which is charged up by a constant current

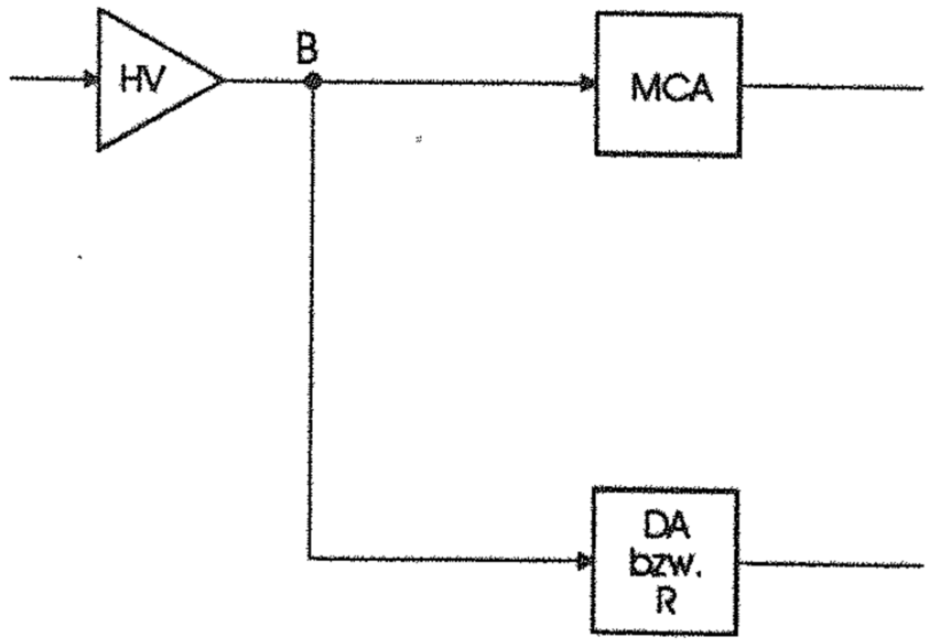

Figure 3.9: Output load of the main amplifier.

during the time interval defined by the start and stop signals. (A nearly constant current is generated by a constant voltage and a large capacitance as long as it is only weakly charged-up.) The capacitor is discharged by a resistor producing a voltage signal proportional to the length of the time interval.

The MCA is connected at point D of Fig. 3.3 to take the time spectrum.

Due to the finite time resolution it may happen that the stop signal arrives before the start signal at the TAC. To avoid this we add a delay of about 200 ns in the stop branch at position C of Fig. 3.3.

4.1 Time Calibration

The TAC has to be calibrated to convert the MCA channel numbers into real time. We perform the calibration with the circuit shown in Fig. 4.1. At point C we connect a pulse generator (or we use one of the SCA outputs and we split the signal, one branch is connected directly to the TAC start, the other signal is delayed using a few passive delay boxes and fed TAC stop. For a fixed delay setting we observe a peak at the MCA. From the linear correlation of the delays with the peak locations we can derive the relation between channel number and the time. This way we obtain the relative scales. To derive the absolute time scale we would have to include in our considerations also the various cables.

Do we need the absolute time scale for the determination of the lifetime?

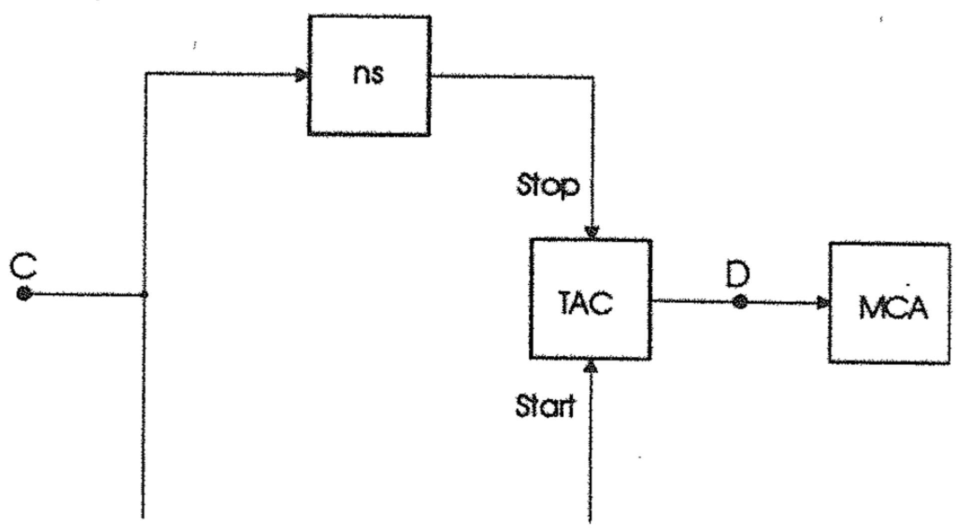

Figure 4.1: Circuit the calibrate the time scale.

5 Experimental Tasks and Data Analysis

5.1 Experimantal Task

1. Sketch the relevant parts of the set-up and indicate its size. Sketch also the signals of the preamplifier, the main amplifier and the linear gate. Measure amplitudes, rise times and pulse lengths. Compare the signals in view of their functions.

2. Take the γ spectrum of the source from both branches. Discuss the spectra and compare the two.

3. Calibrate the apparatus, as discussed above and collect the decay time data. If necessary subtract a constant background. Determine the lifetime of the 14.4 keV state of ^{57}Fe

   - by rough estimate using a logarithmic scale for the channel content,
   - by a fit with a straight line to the logarithmic plot,
   - from the mean value of the lifetimes.

4. Perform a χ^2-test.

5.2 Some Remarks Concerning the Analysis

5.2.1 γ Spectrum

Estimate the energy of the escape peak from known energies of the γ peaks.

5.2.2 Measurement of the Lifetime

Determine the channel-to-time conversion factor by fitting a straight line to the calibration plot.

Due to the finite resolution the observed time t' differs from the real time. This fact can be represented by a resolution function $r(t', t)$, which gives the probability distribution of the measured time t' for the true time t. The measured time distribution $f'(t')$ is obtained from the true distribution $f(t)$ by folding it with the resolution function r.

$$f'(t') = \int_0^\infty f(t) r(t', t) dt \tag{5.1}$$

Discuss the influence of a bell shaped (gaussian) resolution function

$$r(t', t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(t'-t)^2}{2\sigma^2}} \tag{5.2}$$

on an exponantial time distribution

$$f(t) = f_0 e^{-\lambda t} \tag{5.3}$$

Plot the time distribution on a semi-logarithmic paper (or $\log(f)$) as a function of t. Use a binning with no more than 20 bins in total and indicate at some points the statistical error.

Draw a straight line through the linear part of the logarithmic distribution (eye ball fit). Estimate from this best line the lifetime. Add two limiting lines which you think are still compatible with the data to obtain an upper and lower limit of the lifetime.

After this rough estimate we use a more professional method.

We select a part of the distribution where the resolution effects are unimportant (exponential part, or linear in logarithmic representation). We fit straight line to this part of the distribution. To estimate a possible contribution from a constant background, we subtract the result of the fit from the distribution (see Fig 5.3). If there is a sizable background we subtract it from the distribution and repeat the fit.

The most precise value for the lifetime is obtained from its mean value.

$$\tau \approx < t >= \sum_{i=1}^{N} t_i/N \tag{5.4}$$

There are however two problems: i) the deformation of the distribution at short lifetimes and ii) the finite length of the time interval. The first problem is easily solved by shifting the time zero to the beginning of the usable part of the spectrum. (An exponential does not change its shape when we shift the zero). For the loss of long lifetimes $t > t_{max}$ (second problem) we have to correct for. If we knew the true lifetimes $\tau = 1/\lambda$ we could calculate the correction using the relation

$$\tau' = \int_0^{t_{max}} t\lambda e^{-\lambda t} dt \tag{5.5}$$

between the measured mean life τ' and true lifetime τ. Since we do not know τ, we use an iterative procedure. We compute the experimental mean and use our eye ball fit to correct this result. The corrected result is then used to calculate an improved correction a.s.o. until the result does not change significantly anymore.

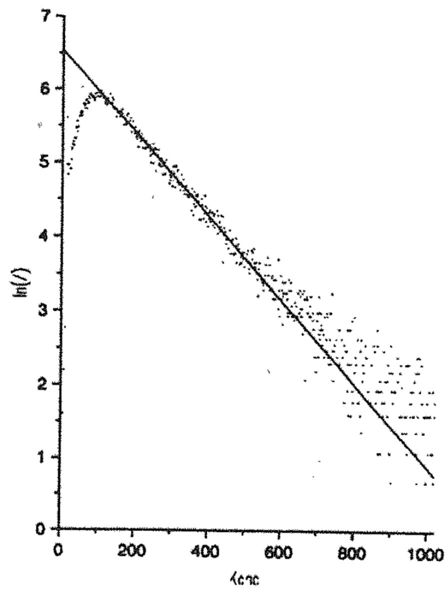

Figure 5.1: Uncorrected decat time distribution.

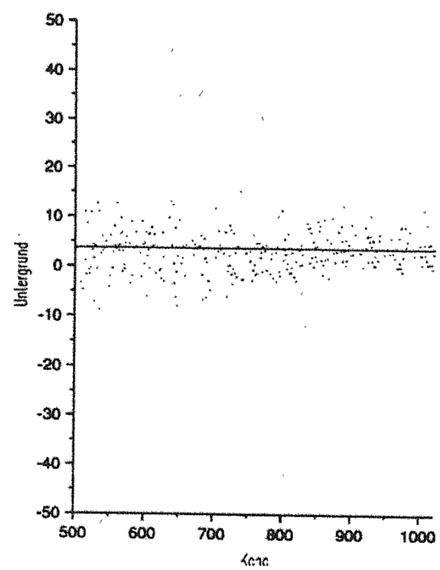

Figure 5.2: Background.

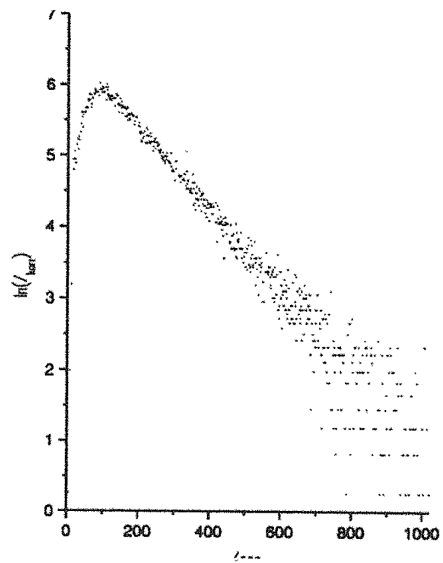

Figure 5.3: Decay time distibution after background subtraction.

6 Appendix

6.1 Poisson Distribution

The number of counts k in a certain time bin follows a Poisson distribution

$$W(K) = \frac{\mu^k}{k!} e^{-\mu} \tag{6.1}$$

where μ is the mean value or expectation value of k. The distribution for different values of μ is shown in Fig. 6.1. The variance σ^2 of the Poisson distribution is

$$\sigma^2 = \mu \tag{6.2}$$

Since for large a large number k of observed events, k is not very different from μ we associate to each bin a statistical uncertainty of $\sqrt{k}$. The relative error is $\Delta k/k = 1/\sqrt{k}$.

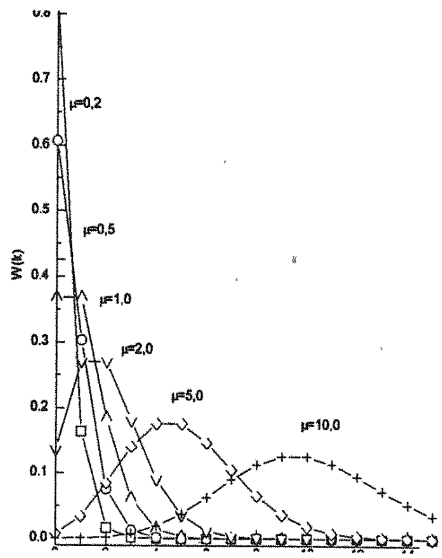

Figure 6.1: Poisson distribution for different mean values. The discrete points are joined together to seperate the distributions.

6.2 χ^2-Test

The χ^2-test is used to compare a measured histogram to a theoretical prediction. Thus it can be used either to check the theoretical assumption or if the theory is well established to identity systematic errors in the measurement. Since we know from very basic quantum mechanical principles and also from many measurements that the decay times of unstable nuclei or particles follow exponential distributions, we use this test to check our experimental procedure.

We histogram our decay times in about 10 to 20 bins and associate to the center of the bin the corresponding event number z_i. Then we normalize the fitted lifetime distribution to the total number of events N_{tot} used in the test:

$$\int_{t_{min}}^{t_{max}} ae^{-t/\tau}dt = N_{tot} \tag{6.3}$$

We compute from this formula the normalization constant and the prediction z_i^{th} for each bin i. Now we want to check how well the prediction coincides with the observation and we compute for each bin a quantity χ_i^2.

The mean squared deviation of the measured number to the predicted number is given by the variance $\sigma^2 = z_i^{th}$ of the Poisson distribution. We normalize the observed quadratic deviation to the expected deviation:

$$\chi_i^2 = \frac{(z_i - z_i^{th})^2}{z_i^{th}} \tag{6.4}$$

Since the denominator is the expectation value of the nominator, the expectation value of χ_i^2 is equal one

$$E(\chi_i^2) = 1 \tag{6.5}$$

The expectation of the sum of all normalized squared deviations

$$\chi^2 = \sum_{i=1}^{B} \chi_i^2 = \sum_{i=1}^{B} \frac{(z_i - z_i^{th})^2}{z_i^{th}} \tag{6.6}$$

is equal to the number B of bins

$$E(\chi^2) = B \tag{6.7}$$

When χ^2 is much larger than B then the agreement of the data with the prediction is bad. To quantify the compatibility, we need the robability density $f_B(\chi^2)$ of χ^2. This function is given in the standard text books of statisrics where we also find tables. We define the so-called χ^2 probability of C. It is fraction of cases (if we perform a large number of identical experiments) where the χ^2 is larger than the specific value of the experiment to be checked.

$$C = \int_0^{\chi^2} f_B(u)du \tag{6.8}$$

If this probability turns out to be for example as small as 2% than we can either conclude we had bad luck to fall into the 2% of cases where the data agree worst with the theory or that something is wrong with the data or the theory.

The χ^2 test is only sensitive to systematic errors when the statistical errors are not too large. Therefore you should not use more than. 10 to 20 bins.

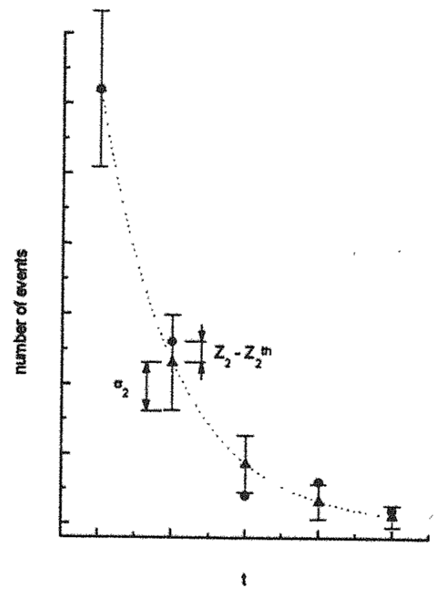

Figure 6.2: Illustration of the χ^2 test.

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