

# **Measurement of the Lifetime of Muons and Pions**

## **Outline of the Experiment:**

Stopping cosmic ray muons and pions are to be measured with a liquid scintillator. The lifetime of these particles is to be determined from their decay time spectra.

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# 1 Experimental Setup

In this experiment the lifetime of elementary particles is inferred from the time delay between the stopping particle and the observation of the decay particles. Cosmic ray particles are stopped in a liquid scintillator tank which is equipped with four photomultipliers. The measurement of decays can be done by using a majority coincidence. The purpose of using this majority coincidence is to suppress noise and chance coincidence signals. It is important to remember that the muon from the  $\pi \rightarrow \mu\nu$  decay gets only a small amount of energy due to the small difference in pion and muon mass. The muon decay ( $\mu \rightarrow e\nu\bar{\nu}$ ), being a three-body decay, leads to a continuous electron spectrum.

The lifetime measurement for muons and pions uses a common electronic logic. However, because of the large differences of lifetimes for pions on the one hand and muons on the other hand, different time calibrations and different time windows for the time-to-amplitude converter have to be chosen. The circuit diagram is shown in Fig. 1. In the preparation of the experiment and during data taking the following items have to be considered:

1. adjustment of threshold and width of the signals at the discriminators,
2. choice of the majority condition at the logic unit,
3. adjustment of the gate width at the gate generator,
4. choice of the time window at the time amplitude converter,
5. correct timing of the signals (of particular importance for the measurement of the short-lived pions).

## 2 Basics of Cosmic Rays and Lifetime Measurement

### 2.1 Cosmic Rays

The existence of a penetrating radiation coming from outer space was demonstrated at the turn of the 19<sup>th</sup> century by measurements on the conductivity of gases. At that time it was believed that gases should be very nearly perfect insulators provided that the applied electric field was not too high.

However, many workers, among them C.T.R. Wilson, Elster and Geitel, showed that in spite of careful precautions to prevent the known radiations from reaching the samples of air in their ionisation chambers, a significant residual conductivity remained. It was found that a reduction in conductivity resulted from shielding the ionisation chambers by lead and this fact was interpreted as showing that most of the residual conductivity was due to an external radiation of some form.

Hess and Kohlhörster investigated the conductivity and ionisation of the atmosphere in 1913. They found that the ionizing radiation increased with increasing altitude in the atmosphere. Therefore, they suggested that this radiation must be of extraterrestrial origin.

The development of cloud chambers and nuclear emulsions enabled the investigation of the relatively complicated interaction processes of cosmic rays. From the fifties on also the use of accelerators helped to identify the various elementary particles and their characteristic interactions. These investigations resulted in the definition of the following components of cosmic rays:

- a) nucleon component
- b) muon component
- c) soft component ( $\beta$  and  $\gamma$ -rays)
- d) neutrino component

One distinguishes between primary cosmic rays which are incident at the top of the atmosphere and secondary cosmic rays produced by the propagation of primary cosmic rays through the atmosphere. Primary cosmic rays originate mainly from supernova explosions or pulsars. Another propagation mechanism was proposed by Fermi in 1949 and this describes the acceleration of interstellar matter by collisions with extended time dependent cosmic magnetic fields. Primary cosmic rays are mainly protons (80-90%), 10-15% are  $\alpha$ -particles, i.e. helium nuclei and there are about 1% nuclei with  $Z \geq 3$ . In addition, primary electrons have an abundance of about 1%. The energies of these primary cosmic rays span a range from a few eV to  $10^{21}$  eV.

Particles which are capable to penetrate the earth's magnetic field collide with nuclei of the earth's atmosphere. Depending on the energy of the primary cosmic ray particle more or less secondary particles or nuclear fragments are produced. These fragments are produced under relatively small transverse momenta and are sufficiently energetic to induce further reactions. This initiates a cascade like process which only stops if the energy of secondary particles is sufficiently low to fall below the threshold for further nuclear processes ( $E < 1 \text{ GeV}$ ).

Even though these secondary particles are produced in the earth's atmosphere by collisions of primary particles all particles produced in this process are called cosmic radiation.

Secondary cosmic radiation at sea level is subdivided into the soft ( $e^\pm, \gamma$ ) and penetrating component, where the penetrating component essentially consists of muons with energies between  $10^8$  and  $10^{12}$  eV. Muons are electromagnetic and weakly interacting particles. Their rest mass is approximately  $207 m_e$ . They have a lifetime of  $2,2 \mu\text{s}$ . The fact that muons are sufficiently energetic and decay only under the weak interactions causes that they can reach sea level.

If one assumes that muons have a velocity close to the velocity of light their average range based on classical mechanics would only be

$$s = c \cdot \tau = 3 \cdot 10^8 \text{ ms}^{-1} \cdot 2,2 \cdot 10^{-6} \text{ s} \approx 700 \text{ m} \quad .$$

# Correct Wiring for $\mu\pi\text{K}$ lab

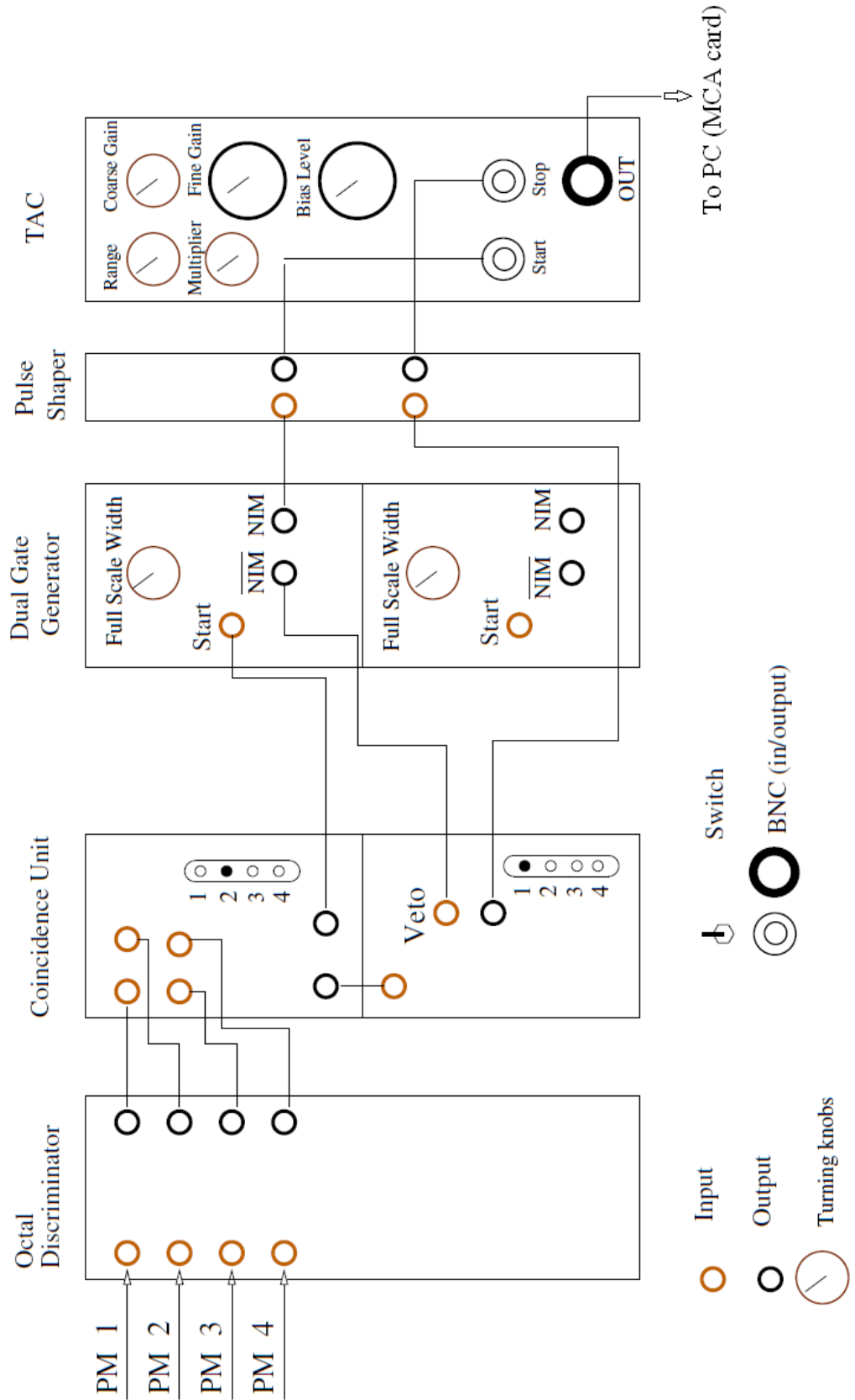


Figure 1: Circuit diagram for the lifetime measurement

Since muons are produced at an altitude of about 15 km this would mean that practically no muon could be recorded at sea level. Actually, one observes a flux of about 1 particle per  $\text{cm}^2$  and minute. This apparent contradiction is resolved if one considers that the lifetime of muons is increased due to relativistic effects because their velocity is rather high and correspondingly their Lorentz factor is very high leading to a lifetime which is

$$\tau = \gamma \cdot \tau_0 \quad .$$

Pions and kaons are produced in the hadronic cascade and also can reach sea level, but also local production by protons, neutrons and muons can provide additional pions and kaons.

The propagation of cosmic rays through the atmosphere is sketched in Fig. 2 and the altitude dependent fluxes of protons, electrons and muons are shown in Fig. 3.

## 2.2 Theory of Special Relativity

The basic input to the theory of special relativity is that the velocity of light is the same in all frames of reference. It is measured to be  $\approx 3 \cdot 10^8$  m/s. The transformation from one frame of reference to a different frame of reference moving at a certain velocity  $v$  is provided by the Lorentz transformation. It is convenient, to introduce the velocity  $\beta$

$$\beta = \frac{v}{c}$$

i.e. the velocity measured in units of the velocity of light. In addition one needs the so-called Lorentz factor  $\gamma$  which is

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad .$$

The relation between the momentum  $p$  of a particle and its total energy is given by

$$E = c \cdot \sqrt{p^2 + m_0^2 \cdot c^2}$$

or equivalently

$$E = \gamma m_0 c^2 = mc^2 \quad .$$

This last equation says that the mass of a particle is not constant but rather is velocity-dependent leading to

$$m = \frac{m_0}{\sqrt{1 - \beta^2}} = \gamma m_0 \quad .$$

Therefore, an acceleration of a particle which is moving already close to the velocity of light leads to an increase of its mass rather than an increase of its velocity. In a similar way it can be shown that the time distance between two

events in a certain frame of reference is different when viewed from a frame of reference moving at a velocity  $v$  with respect to the first frame. Namely,

$$\Delta t' = \gamma \Delta t \quad .$$

The time interval is longer in the moving frame. This result is called time dilatation. Time dilatation is a direct consequence of special relativity.

The time interval measured in the frame where the clock is at rest is called the proper time. Time intervals measured in other frames are always longer.

It must be kept in mind that the energy which is related to the rest mass by  $\gamma m_0 c^2$  is always the total energy of a particle (including its rest mass). The kinetic energy of a particle is

$$E_{\text{kin}} = E - m_0 c^2 \quad .$$

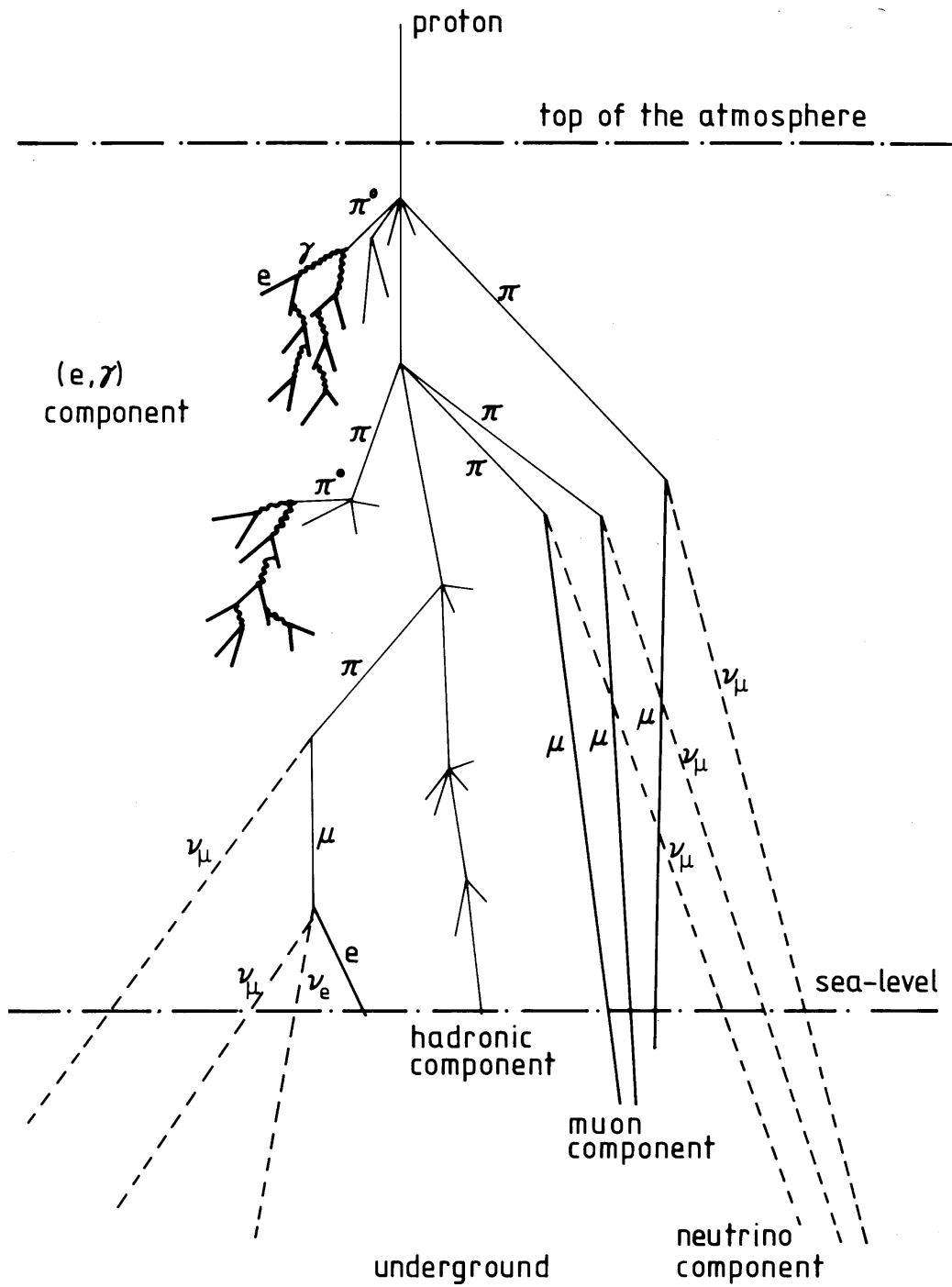


Figure 2: Propagation of cosmic rays in the atmosphere

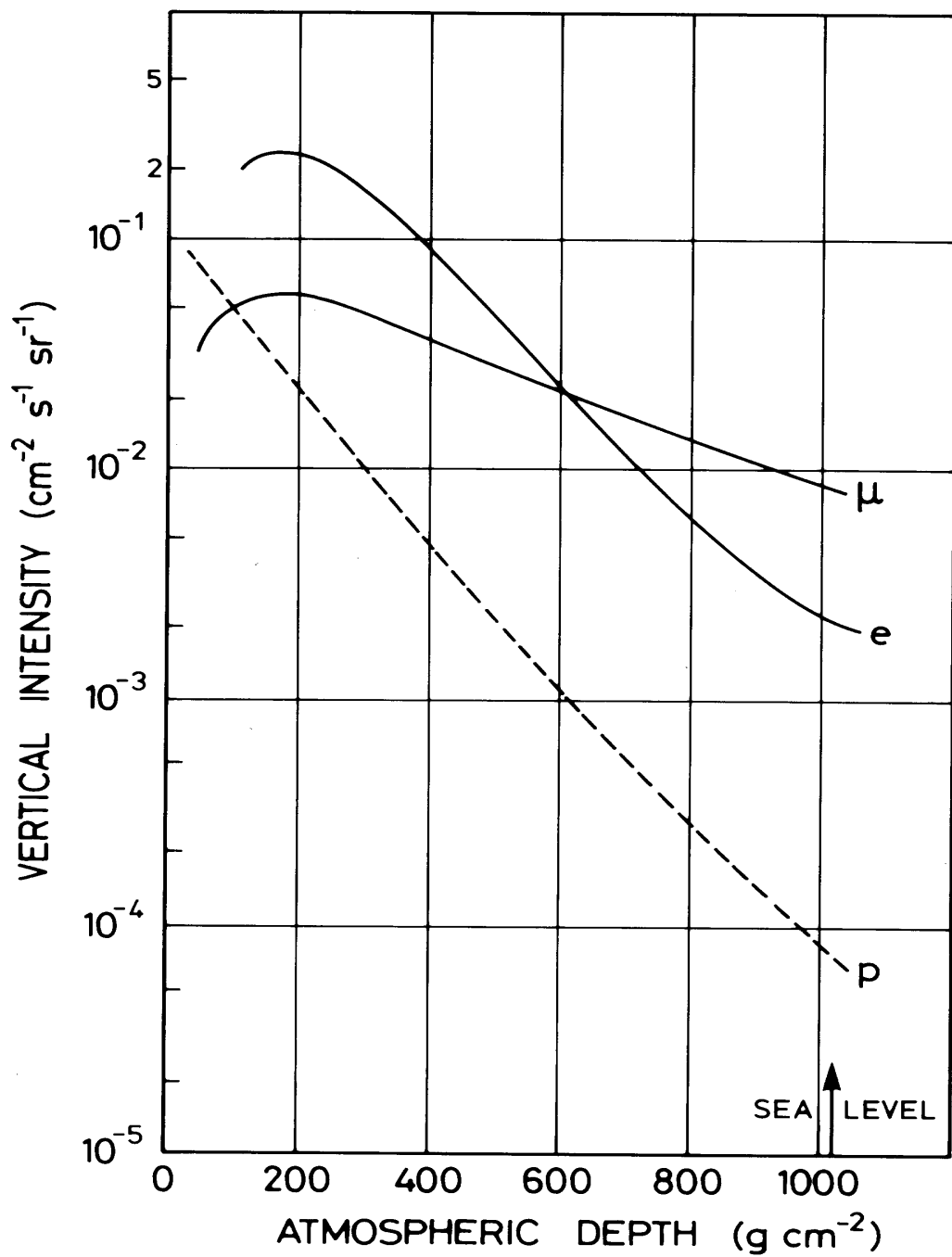


Figure 3: Altitude dependence of muon, proton and electron fluxes in the atmosphere



## 2.3 Physics of Muons

Muons are leptons. Just as electrons they are subject to weak and electromagnetic interactions. Muons carry an angular momentum of  $\frac{1}{2}\hbar$  and are consequently fermions.

Muons were discovered by Anderson and Neddermeyer in 1937. Originally it was suspected that the muon could be the particle postulated by Yukawa which was supposed to be responsible for the strong interactions which bind nucleons in a nucleus together. Only after the discovery of the pion in 1949 it became clear that pions and muons are different particles.

There exist negative and positive muons – particle and antiparticle – with charge  $(-e)$  or  $(+e)$ , respectively. The dominant source of muons is pion decay, but muons are also produced in the decay of other mesons like kaons. Muons decay with an average lifetime of  $\tau = 2.2\mu\text{s}$  into an electron ( $e^+$  or  $e^-$ ) and 2 neutrinos. Since 1962 one knows that these two neutrinos are different particles. They are called the muon neutrino and electron neutrino ( $\nu_\mu$  and  $\nu_e$ ).

The muon decay is subject to several conservation laws like charge conservation and lepton number conservation, where the lepton numbers are independently conserved for the muon and the electron species. Of course also energy and momentum conservation holds.

### Characteristic Properties of Muons and Pions

a) Muons

$$\begin{aligned} \text{Rest mass: } m_\mu &= 105.658389 \text{ MeV}/c^2 \\ \text{Charge: } q &= \pm 1.602 \cdot 10^{-19} \text{ As} \\ \text{Spin: } s &= \frac{1}{2}\hbar \\ \text{Mean life: } \tau_\mu &= (2.19703 \pm 0.00004) \cdot 10^{-6} \text{ s} \end{aligned}$$

b) Pions

$$\begin{aligned} m_{\pi^\pm} &= 139,56995 \text{ MeV}/c^2 \\ \text{Mean life: } \tau_{\pi^\pm} &= 2.6033 \pm 0.0005) \cdot 10^{-8} \text{ s} \\ \text{Spin: } s &= 0 \\ m_{\pi^0} &= (134.9764 \pm 0.0006) \text{ MeV}/c^2 \\ \text{Mean life: } \tau_{\pi^0} &= (8.4 \pm 0.6) \cdot 10^{-17} \text{ s} \\ \text{Spin: } s &= 0 \end{aligned}$$

### Formation Processes of Muons and Decay Processes of Pions

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu && \text{with a branching ratio of nearly} \\ &&& 100\% \\ \pi^- &\rightarrow \mu^- + \bar{\nu}_\mu && 100\% \end{aligned}$$

### Decay Processes of Muons

$$\begin{aligned} \mu^+ \text{ decay: } &\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \\ \mu^- \text{ decay: } &\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \\ \mu^- \text{ capture: } &\mu^- + p \rightarrow n + \nu_\mu \end{aligned}$$

The decay processes of muons are three-body decays, which means that the spectra of decay electrons are continuous in a similar way as the decay spectra of electrons in nuclear beta decay. The maximum transferable energy to the electron  $E_{e,\max}^\mu$  is approximately half the rest mass of the muon corresponding to 53 MeV. The electron spectrum of a muon decay at rest is shown in Fig. 4.

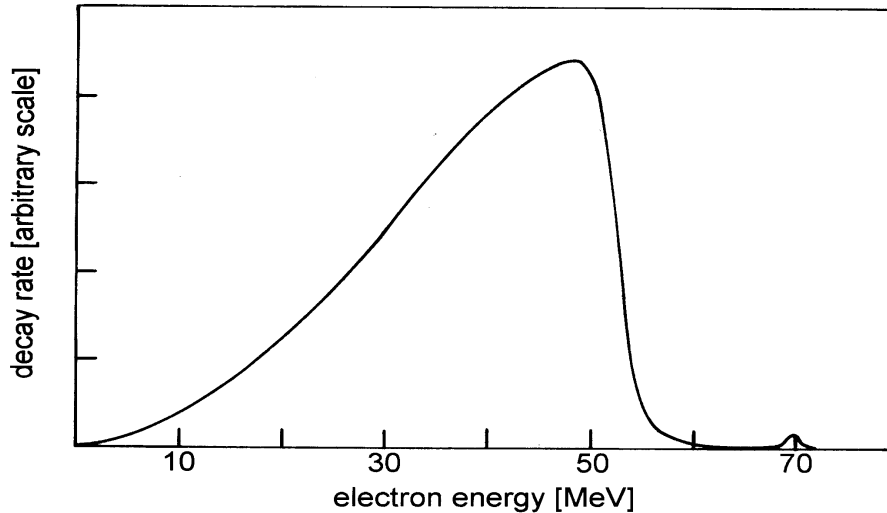


Figure 4: Energy spectrum of electrons from muon decay at rest

In competition to muon decay negative muons can also be captured by nuclei in a similar fashion as electrons from the K-shell can be captured by a nucleus. The probability for a muon capture increases with the fourth power of the nuclear charge number.

Muons can be captured into the K-shell of an atom forming a muonic atom. Since the rest mass of muons is larger by a factor of about 200 compared to electrons their orbit is approximately a factor of 200 closer to the nucleus. Consequently, there is a non-vanishing overlap of the wave function of the muon with the wave function of the nucleus and this can lead to a capture of the negative muon by the nucleus. For  $Z \geq 40$  the Bohr orbit is already inside the nucleus. The capture probability for this condition is very close to 1. For light nuclei, however, with charge number of  $\approx 10$  or less, also negative muons will preferentially decay in an analogue fashion to  $\mu^+$  decay.

The decay probability for free muons at rest  $\lambda = \frac{1}{\tau}$  is the same for positive and negative muons.

$$\lambda^+ = \lambda^- \quad \text{or} \quad \tau^+ = \tau^-$$

If negative muons are stopped in matter the measured decay probability  $\lambda_m^-$  is always larger than the decay probability  $\lambda^+$  or  $\lambda^-$  for free muons. So the equation

$$\lambda_m^- > \lambda^+ = \lambda^-$$

holds or, respectively

$$\tau_m^- < \tau^+ = \tau^- \quad .$$

In this experiment muons are stopped in a liquid scintillator. The scintillation material consists essentially of carbon. The mean life of negative muons in carbon

has been measured to be  $2.026 \mu\text{s}$ . For light elements, e.g. hydrogen or helium, the mean life is equal to the lifetime of a free muon =  $2.20 \mu\text{s}$ .

## 2.4 Scintillators and Photomultipliers

Muons and pions are stopped in a liquid scintillator. This scintillation counter is a mixture of three different components: the primary scintillator, the wavelength shifter and the base material. Certain types of molecules will release a small fraction of the absorbed energy as optical photons. This process is especially marked in organic substances which contain aromatic rings such as polystyrene, polyvinyltoluene and naphthalene. Liquids which scintillate include toluene or xylene.

This primary scintillation light is preferentially emitted in the UV-range. The absorption length for UV-photons in the scintillation material is rather short. The scintillator is not transparent for its own scintillation light. Therefore, this light is transferred to a wavelength shifter which absorbs the UV-light and reemits it at longer wavelength (e.g. in the green). Due to the lower concentration of the wavelength-shifter material the emitted light can get out of the scintillator and be detected by a photomultiplier. The technique of wavelength shifting is also used to match the emitted light to the spectral sensitivity of the photomultiplier. For plastic scintillators the primary scintillator and wavelength shifter are mixed with organic material to form a polymerizing structure. In liquid scintillators the two active components are mixed with an organic base. About 100 eV are required to produce one photon in an organic scintillator.

In this experiment the liquid scintillator consists of p-terphenyle as primary scintillator, POPOP as wavelength shifter and Uvasol as base material. The readout is accomplished by four multiplier tubes which are coupled to the liquid scintillator via an air light guide.

## 2.5 Determination of the Mean Life

The law of radioactive decay

$$N = N_0 e^{-\lambda t}$$

is a statistical law which means that it is valid for many different decays. There is no way to predict the decay of an individual particle. In particular the decay probability does not depend on the fact that a certain particle may have lived already for a certain time. The measurement of the mean life of muons and pions is based on the following principle:

The deceleration of a muon or pion in the liquid scintillator gives rise to a light signal which is recorded by the photomultipliers. This is used as a trigger for the time-to-amplitude converter. When the muon (or  $\pi$ ) has come to rest no further light is emitted. Only the electrons (or decay products of pions) which appear after muon decay provide a second signal at a later time which stops the time-to-amplitude converter. The time difference between the incoming, i.e. stopping muon and the appearance of the electron is a measure for the average lifetime or the decay time of individual muons.

By summing up many events of the same kind one obtains a time spectrum of decay times which can be parametrized by the law of the radioactive decay.

If muons decay in flight, the muon signal cannot be distinguished from the electron signal and consequently there will be no stop at the input of the time-to-amplitude converter. Therefore, muons decaying in flight are simply ignored by the circuitry. Signals caused by radioactive background are rare and can be suppressed by adjusting the discriminator threshold in an appropriate fashion.

Since muons in cosmic rays are a mixture of positive and negative muons one will observe three different components in the time spectrum as measured by the time-to-amplitude converter. The decay of free positive muons, the decay of negative muons and chance coincidences.

Chance coincidences can be measured at very large decay times where practically all muons are decayed. The contribution of chance coincidences is usually small but they have to be taken into account. They are uniformly distributed over all times.

Since the decaying positive and negative muons have a different lifetime (due to muon capture for negative muons), the correct fit function would be the sum of two exponentials with individual lifetimes - since the sum of two exponentials with different slopes is not an exponential again. The lifetime of the negative muons can be fixed to  $2.026 \mu\text{s}$ , the value for stopping negative muons in carbon. Further we can make use of the fact that there is an excess of positive muons at sea level, the ratio between positive and negative muons is constant over a wide momentum range and takes a value of

$$R = \frac{N(\mu^+)}{N(\mu^-)} \approx 1.27 \quad .$$

Details about the fitting function can be found in chapter 4.

In a similar fashion the spectrum recorded for pion decay contains four different contributions: The first contribution comes from the decays of kaons ( $K^+$  with a lifetime of 12.4 ns). The kaons will dominate the spectrum at short decay times. The second contribution comes from chance coincidences which supplies a uniform distribution. The third distribution comes from muon decay which is also rather uniform over a time range of 200 ns because the lifetime of muons is rather long compared to the lifetime of pions. (For this correction one can use an average effective common lifetime for  $\mu^+$  and  $\mu^-$ .) The fourth contribution finally comes from  $\pi^+$  decay ( $\pi^-$  are always captured by nuclei). In analyzing the pion decay spectrum one has to take into account chance coincidences, muon decays, and kaon decays. Electronic noise affects the measurement at very short decay times (below  $\approx 45$  ns), thus this part of the spectrum should not be used for the fitting process, see chapter 4 for details.

### 3 How to Do the Experiment

The negative high voltage is connected to the photomultipliers, it should never exceed  $\approx -1.9$  kV.

The light from the liquid scintillator is measured with four photomultipliers. The start and stop signals can be derived from a majority condition. In princi-

ple the majority condition depends on the background, the chosen discriminator thresholds and the high voltage. In pion decay muons get only very little kinetic energy so that a soft majority condition should be used for the measurement of pion decays. Since the electrons in muon decay can get a rather large kinetic energy, a more stringent majority condition could be selected to reject background. However, it has been found that a majority condition of “2 out of 4” gives good results in both cases.

The circuit diagram for the measurement of the mean life of muons and pions is shown in Fig. 1. The gate generator defines the time window in which stop signals can be accepted. In the same way the gate generator inhibits start signals which occur during this time window.

1. Observe the signals of the photomultipliers on the oscilloscope and sketch their signal shape.
2. Observe the logic signals after the discriminators and adjust the threshold of the discriminators in such a way that muons and pions and their decay products can be measured.
3. Check the wiring of all NIM modules and also check the proper functioning of the circuit by looking at the signals in different places using the oscilloscope. Use the pulse generator which supplies signals of known width and known relative delay. Make a sketch of all signal shapes at all important points of the electronic circuitry.
4. For the measurement of muons one should select a time window on the gate generator of the order of  $10 \mu\text{s}$  while for the measurement of pions the time window should be of the order of 200 ns.
5. Describe and motivate the electronic logic for the measurement of the decay times of muons and pions.

## 4 Data Analysis

The data are recorded by a multichannel analyzer and can be graphically displayed on a screen.

1. Determine the lifetime of muons. The recorded data are to be corrected for instrumental effects like background. Start with the simple fit function

$$N = N_0 e^{-t/\tau} + b$$

assuming that positive and negative muons have the same lifetime. Then use the reduced lifetime of stopping negative muons (due to muon capture) and the ratio of negatively and positively charged muons as given in section 2.5 for a modified fit function

$$N = N_0(e^{-t/\tau_{\mu^+}} + (1/1.27) e^{-t/2.026}) + b$$

Compare and discuss the two results.

- Determine the lifetime of pions. The decay spectrum of pions has to be corrected for chance coincidences and kaon and muon decay. Determine the pion mean life from a fit

$$N = N_0 e^{-t/\tau_\pi} + b$$

to the data for time differences where one can be sure that the kaon decay can be neglected (e.g. larger than  $\approx 70$  ns). The contributions from chance coincidences and muon decay are taken as a uniform background  $b$  (see chapter 2.5).

The lifetime of the kaons is so short that it cannot be determined very precisely with our experimental setup. One can make an exponential fit like for the pions in a small time interval (45 - 60 ns). Another option is to use a double exponential fit function

$$N = N_{0K} e^{-t/\tau_K} + N_{0\pi} e^{-t/\tau_\pi} + b$$

with independent parameters for kaons and pions and fit over the full time range (excluding very short decay times  $< 45$  ns) . Compare and discuss the results of the different fitting functions.

- Determine statistical and systematical errors. Check how variations of the lower and upper fit-limits change the results of the fit.
- Plot all data in a graphical way (on linear and semi-logarithmic scales).
- Determine the rate of muons and the rate of pions which stop in the detector. To estimate the numbers of muons and pions, use the fit function  $N(t) = N_0 e^{-t/\tau} + b$  you derived in the steps before. The area under the exponential function gives a good estimate for the number of particles. Integrating the exponential function from 0 to  $\infty$  gives  $N_{tot(t)} = N_0\tau$ . Now one just needs to divide this number by the slope of the calibration (since the spectrum was derived in channels) to get the number of muons or pions.

## Problems

- The mean life of muons at rest is  $\tau = 2.2 \mu\text{s}$ , their rest mass is  $m_0 = 207 m_e$ . What is the kinetic energy of a muon produced at an altitude of 20 km which is required to reach sea level.
- Determine the kinetic energy of muons from pion decay at rest.
- Explain the small bump at 70 MeV in Fig. 4.

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## 6 Short Description of How to Operate the Multi-Channel Analyzer

The measurement is done using a Multichannel Analyzer Card ‘MCA-3A’ installed in a personal computer running Windows 10.

The provided ‘MCDWIN’ software allows to produce a linear or logarithmic plot of the time spectrum, perform gaussian fits to the calibration spectrum, select data intervals using the mouse, and so on. The supervisor will show you how to do the measurements using the ‘MCDWIN’ software. Details of the software are provided in the manual

<https://www.fastcomtec.com/ftp/manuals/mca3doc.pdf>

(See chapter 5 for details).

The data is saved on the computer or on an USB-Stick in ASCII format, so that the analysis can be done with your home computer.