Monte Carlo Tools in Particle Physics

Master Laboratory Course (2 Days)

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Contents

Introduction

During this laboratory course you will be introduced to the fundamentals of modern particle physics by developing a basic understanding of general purpose Monte-Carlo event generators (MCEGs) [1].

In particular, you will be introduced the `WHIZARD` [2] event generator which is a sophisticated tool for automatic simulations of particle collisions and decays.

Before any simulations a basic understanding of particle physics and Monte-Carlo Methods is required: This will be covered in the chapters 1.1.1 and 1.1.2. Furthermore the *Standard Model of particle physics* (SM) will be presented in Chapter 1.1.1 focusing on the *electroweak force* later in Chapter 2.1.2. The basics of Monte Carlo integration and its application to event generation will be discussed in 1.1.2.

You will be introduced to the concepts of particle scattering and particle decays. Moreover you will learn how such processes are calculated in a *quantum field theory* (QFT) in Chapter 2.1.1. From this you can see the need for MCEGs such as `WHIZARD` , a MCEG that can calculate such objects numerically as the analytic expression grows factorially.

Since installing `WHIZARD` , or any other particle physics software, is a bit sophisticated the installation of `WHIZARD` is part of the preparation for this course.

On the first day (part 1) you will study and manipulate a given simulation setup in order to understand the syntax of `WHIZARD`'s scripting language `sindarin` and develop basic understanding of particle reaction simulations.

The second day will be reserved for writing your own simulation scripts. More details will be provided in part 2.

1 First Day

1.1 Prerequisites

1.1.1 The Standard Model of Particle Physics

The *standard model of particle physics* (SM) [3–5] is a relativistic quantum field theory which describes three of the four known fundamental interactions. It includes three generations of *quarks* and *leptons* which both are fermions. Together they are the constituents of all visible matter. Furthermore the SM contains *vector bosons* which act as the mediators of all represented fundamental interactions. Finally one *scalar boson* the Higgs-boson h is included in the Standard Model. It takes part in the unification of the weak and electromagnetic interaction and gives masses to all SM particles.

The *strong interaction* is responsible for the binding of quarks to hadrons, like neutrons and protons, as well as the cohesion of the atomic nucleus. This interaction is mediated by the so called *gluons* g which are massless bosons. This interaction will not be part of this course.

The well known *electromagnetic interaction* which is mediated by the massless *photon* γ is also included in the Standard Model. It mediates interactions between electrically charged particles.

The *weak interaction* is the force behind radioactive decays of matter. Its mediators are the massive $W^{\pm}$ and Z-bosons. In comparison to the electron these bosons are very heavy so $m_W = 80.379 \pm 0.012$ GeV[1][2] and $m_Z = 91.1876 \pm 0.0021$ GeV. Moreover the Higgs boson's mass is about the same order of magnitude $m_h = 125.15 \pm 0.17$ GeV.

The electromagnetic and weak interaction are unified in the *electroweak interaction* in which the Higgs-boson h plays an important role. In this sophisticated mechanism h is expanded around its vacuum expectation value[3] and give mass to all SM particles. Quarks, leptons as well as the gauge bosons of the weak interaction are affected by it. This course evolves around this interaction. Further information about it will be given in 2.1.2.

The *gravitation* –the last of the four fundamental interactions– is not included in the Standard Model.

The Higgs-boson also interacts with other particles and couples to all SM particles proportional to their masses. The SM's charged leptons –the electron e^-, the muon μ^- and the tau τ^-–which all have corresponding anti-particles with opposite charge. Their anti-particles are the positron e^+, the anti-muon μ^+ and the anti-tau τ^+. The charged lepton's masses arrange over several orders of magnitude. From the electron $m_e = 0.5109989461 \pm 0.0000000031$ keV over the muon $m_\mu = 105.6583745 \pm 0.000002$ MeV to the tau $m_\tau = 1776.86 \pm 0.12$ MeV.

Each charged lepton comes with a corresponding neutrino (ν_e, ν_μ, ν_τ). Neutrinos are electrically uncharged particles only interacting via the weak force. They all have very low masses $m_\nu < 2$ eV. Like all other SM fermions the neutrinos also have corresponding anti-particles ($\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$).

The second group of SM fermions are the quarks which come in six different *flavours*. These include the up u, strange s and bottom b quark. All having a fractional charges of $2/3$. The remaining three quarks, with charge $-1/3$ quarks are the down d, charm c and top t. The quark

[1] Note that we use natural units is particle physics so the speed of light c, the Plank-constant $\hbar$, the gravitational constant G and the Bolzmann-constant k_b are $c = \hbar = G = k_b = 1$. This leads to the mass having the unit of the energy. In SI-units the mass would have $[m] = \frac{\text{GeV}}{c^2}$. In these units the mass of the proton is $m_p \approx 1$ GeV.

[2] All numeric values can be found in the *Particle Data Book* (PDB) [6].

[3] You may think of this as the expansion around its degenerated minimal values.

masses also arrange over several orders from $m_u \approx 2$ MeV to $m_t = 172.76 \pm 0.3$ GeV which is the heaviest particle in the Standard Model. Each quark has an associated anti-particle which again has almost the same properties but the opposite electric charge.

The SM is mostly used to calculate so called *cross sections* of particle collision and *decay widths* of particles. Booth are Lorentz-invariant measures of a possibility for a particle reaction to happen. Calculating them involves solving often complicated multidimensional integrals in the relativistic phase space 2.1.1. In this course you will use Monte-Carlo techniques explained in Chapter 1.1.2 to solve those integrals.

Particle Family	Particles	C	EM	weak	strong
charged leptons	e^-, μ^-, τ^-	-1	x	x	
neutrinos	ν_e, ν_μ, ν_τ	0		x	
Up Quarks	u, c, t	$2/3$	x	x	x
Down Quarks	d, s, b	$-1/3$	x	x	x

Table 1: Information about the interactions SM fermions are involved in. C is the fermions charge and 'x' means involved in this interaction. Note that each fermion has a corresponding anti-fermion which opposite charge.

1.1.2 Monte-Carlo Techniques

Monte-Carlo: A first Example

As a first example we apply a Monte-Carlo approach for calculating the value of the irrational number π. For that we consider a cycle with radius $r/2$ inside of a square with length r. The ratio R we define

$$\mathrm{R} = 4 \cdot \frac{A_{\text{circle}}}{A_{\text{quad}}} = 4 \cdot \frac{\left(\frac{r}{2}\right)^2 \pi}{r \cdot r} = \pi \quad . \tag{1.1}$$

Since this ratio R is independent from Radius r we can choose $r = 1$ from now on.

Now we generating N pairs of random numbers x_i and y_i with $x_i, y_i \in [0, 1]$. M shell be the number of pair matching

$$x_i^2 + y_i^2 \leq 1 \quad . \tag{1.2}$$

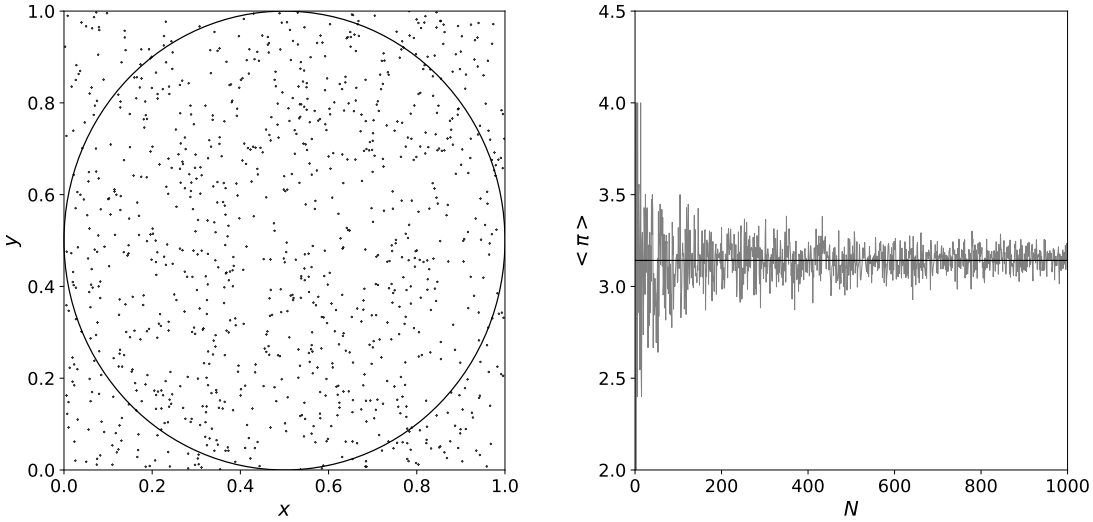
For a visualization of this condition see figure 1(a).

Hence M is number of points being inside of the cycle and N is the number of points in the complete square.

According to 1.1 π simply can be estimated by

$$\langle \pi \rangle = 4 \cdot \frac{M}{N} \quad . \tag{1.3}$$

The convergences for this algorithm is shown in figure 1(b).



(a) Scattering of the random numbers

(b) Convergences of the hint-or-miss algorithm

Figure 1: (a) Distribution of $N = 1000$ random number pairs (x_i, y_i). (b) Estimation $\langle\pi\rangle$ in dependence the number of iterations of the presented algorithm N. The gray line belongs to the estimation the black line corresponds to the real value of π.

Monte-Carlo Integration

Since evaluating –especially multidimensional– integrals can be a tedious task various deterministic and non-deterministic methods have been developed to calculate them. One of them is the so called *Monte-Carlo* integration [7]. This is a non-deterministic numeric technique that uses random numbers to approximate the value of an integral. Due to it being non-deterministic the outcome of each integration is different.

First consider a n-dimensional real integral I of the regular function $\mathrm{f}(\vec{x}) < \mathrm{f}_{\max}$

$$\mathrm{I} = \int_\Omega \mathrm{d}\vec{x}\, \mathrm{f}(\vec{x})\,, \quad \Omega \subset \mathbb{R}^n \quad . \tag{1.4}$$

Obviously,

$$\mathrm{I} < \mathrm{f}_{\max} \cdot \int_\Omega \mathrm{d}\vec{x} := \mathrm{V} \quad . \tag{1.5}$$

Now consider each $\vec{x}_i \in \Omega$ as vector of (pseudo) random numbers[4]. Therefore we can approximate

$$\mathrm{I} \approx \mathrm{P}_N := \frac{\mathrm{V}}{N} \sum_i^N \mathrm{f}(\vec{x_i}) = \mathrm{V} \cdot \mathrm{E}[\mathrm{f}] \quad . \tag{1.6}$$

[4]Pseudo random numbers are produced algorithmically from so called Markov-Chains and therefor depend on an initial value which is called *seed*.

Due to the *law of large numbers*

$$\mathrm{I} = \lim_{N\to\infty} \mathrm{P}_N \quad . \tag{1.7}$$

Therefore the convergences of the integral is ensured for a sufficiently large number N.
By using the some properties of the variance $V[P_N] = \sigma^2_{P_N}$ you might show that

$$\sigma_{\mathrm{P}_N} \sim \frac{1}{\sqrt{N}} \quad . \tag{1.8}$$

Note that the integration's accuracy therefor is dependent on the number of iterations N and the scaling of its standard derivation σ_{P_N} can be estimated by $\frac{1}{\sqrt{N}}$. However, this cannot be understood as a strict error bound since a particular realization of $\mathrm{P_N}$ may not cover all important features of an integral.

In contrast to many other integration methods the error of the Monte-Carlo integration does not scale with the integral's number of dimensions. This may be its most significant advantage over deterministic integration methods.

1.1.3 WHIZARD

`WHIZARD` [5] [2] is a general purpose MCEG designed for the efficient calculation of multi-particle scattering cross sections and simulated event samples.

The program automatically computes complete *tree-level* matrix elements, integrates them over phase space, evaluates distributions of (kinematic) observables like the invariant mass or the scattering angle.

There is no conceptual limit on the process complexity; using current hardware, the program has successfully been applied to scattering processes with up to eight particles in the final state.

The standard model of particle physics as well as many other models have been implemented.

`WHIZARD` 1.1.4 will be used for all calculations in this course. You will be guided through the installation of `WHIZARD` in chapter and run and investigate a first example on the first day.

1.1.4 Install WHIZARD

Since installing `WHIZARD` might be a non trivial task –especially for someone not familiar with `Linux`– the installation will is explained in this chapter. You are expected to install `WHIZARD` on your own laptop as preparation for the first experiment so that at least one participant has a functional `WHIZARD` installation on a notebook. If the installation is for some reason is not possible on your computer or you don't have a notebook you will find `WHIZARD` installed on a computer at the university and you can also do the installation on your own there.

However installing `WHIZARD` is part of this course so you might at least try it on your own. For further information on the `WHIZARD` installation see.[6]

Note that running `WHIZARD` on `Windows` is not supported. Therefor you need to set up a `Linux` installation first. For that you can set up a `Virtualbox` with a with `Linux` distribution,

[5] `https://whizard.hepforge.org/`
[6] `https://whizard.hepforge.org/manual/manual003.html`

Ubuntu LTS or Debian for example would be sufficient.[7][8] If you are Running Windows 10 you might alternatively set up its Linux subsystem (WSL) supported by Microsoft and install WHIZARD there[9]. Furthermore you can install a second OS on your computer and run it in dual boot.

Packages Linux on Debian that are required for WHIZARD are at least:

- gfortran
- libtool
- gcc and g++
- ocaml
- latex e.g. texlive and texlive-metapost if plots from WHIZARD are desired
- building-essential

Depending on your Linux distribution some further packages might be required. The name of your package manager depends on your Linux distribution. On Debian its name is apt-get.

To install WHIZARD on your own computer you can download it here[10].

Open a terminal on your computer and go to the directory where the installation file is located.

The file is called whizard-3.x.x.tar.gz. The file ending tar.gz means that it's a wrapped bundle of files. It consumes less space and it is easier to transport. First you need to *untar* it.

```
<path>\$ tar -xzfv whizard-3.x.x.tar.gz
```

This command will unpack the files to a folder called whizard-3.x.x. In it you will find a file called makefile.

You can understand it as a recipe for building WHIZARD . It contains all necessary steps to build the functional program.

Create the folders build and install next.

```
<path>\$ mkdir build install
```

The install is the folder where you can find the executable WHIZARD after the complete installation. The folder build will be used to install WHIZARD from within it. Now so change to it.

```
<path>\$ cd build
```

Inside the build folder

```
<path>\build\$~../whizard-3.x.x/configure --prefix=<path>/install/
```

[7] download https://www.virtualbox.org/wiki/Downloads
[8] Set up Virtualbox with Ubuntu: https://www.wikihow.com/Install-Ubuntu-on-VirtualBox
[9] Choose manual WSL install: https://docs.microsoft.com/en-us/windows/wsl/install-win10
[10] https://whizard.hepforge.org/downloads/

This will set up everything for the installation. You don't need any additional options for this course but if you like to use WHIZARD later in a more advanced setting you might specify something here. See the manual for more details here[11]. `--prefix` is used the specify your installation directory. Otherwise WHIZARD would be installed directly to the `build` folder.

Next you the need to install the program.

```
<path>\build\$ make && make install && make check
```

This will execute all three command after the previous command has fully executed without any error. `make` will prepare the installation, `make install` will do the installation and `make check` will check if everything is installed correctly. Note that this will take some time. If `make check` dose not encounter any fails (xfails are fine) you have installed WHIZARD successfully.

1.2 Exercises

After the installation is done you will find a folder `\share\ whizard\ examples` inside the `install` folder. The examples all end with `.sin`. They are called `sindarin` files since `sindarin` is WHIZARD own scripting language. More information about WHIZARD's scripting language `sindarin` can be found here[12].

Now create a new folder for your first WHIZARD run and copy `Z-lineshape.sin` to it. Open the file `Z-lineshape.sin` and study the commands. You might understand a few lines without knowing anything about `sindarin` .

Then run WHIZARD inside that your newly created folder and have a look at the output files.

```
<path>\test_run\$ <installation-path-of-whizard>/whizard Z-lineshape.sin
```

Find the results of your calculation. Does the size of the errors fit your expectations? Find the *seed* of your run. Study the file and try to find the meaning for each command.

Complete the following tasks each one starting from `Z-lineshape.sin`:

A1. First add the error value to your output data then remove the integration of `realcorr`. Now lower the number of iterations for `bornproc` to $2 : 250 : "gw", 1 : 100$ and then further to $1 : 10 : "gw", 1 : 5$. Document your results for booth runs. Compare your results.

A2. Remove all `cuts` from the `sindarin` file.

A3. Again remove the integration of `realcorr` and set the Z-Boson's mass $M_Z = 100$ GeV. Adapt `scan` and `integration` to these changes.

A4. Remove the integration of `realcorr` again and set the Z-Boson's width $W_Z = 5$ GeV. Adapt `scan` and `integration` to these changes.

Document and describe your results in your report. Do your results fit your expectations? Does the error of your results fit your expectation? Furthermore describe the meaning of the changes you made in the `sindarin` files and adapt the plot description. Make sure that your `sindarin` files don't contain artifacts form previous runs and comment them reasonable and meaningfully.

[11] `https://whizard.hepforge.org/manual/manual003.html`
[12] `https://whizard.hepforge.org/manual/manual005.html#sec66`

```
# We choose our favourite model
model = SM

# Define incomming particle beam
beams = e1, E1

# Define some particle containers for the cuts
alias lep = e1:E1:e2:E2
alias prt = lep:A

# These are the two processes we want to compare
process bornproc = e1, E1 => e2, E2
process realcorr = e1, E1 => e2, E2, gamma

# Compile model and process information
compile

# This is a cut on the phase space. If 'true' the matrix element will be set to zero

cuts = all E >= 100 MeV [prt]
   and all abs (cos(Theta)) <= 0.99 [prt]
   and all M2 >= (1 GeV)^2 [prt, prt]

# Define title and labels as global variables that will be used in the plot:
$description = "A WHIZARD 3.0 Example"
$x_label = "$\sqrt{s}$/GeV"
$y_label = "$\sigma(s)$/pb"
x_min = 88 GeV
x_max = 95 GeV

# Allocate one plot
$title = "The Z Lineshape in $e^+e^-\to\mu^+\mu^-$"
plot lineshape_born

# Allocate another plot
$title = "The Z Lineshape in $e^+e^-\to\mu^+\mu^-\gamma$"
plot lineshape_realcorr {$draw_options = "withcolor blue"}

# Compute the cross sections for different sqrts
# with smaller steps around the peak
scan sqrts = ((88.0 GeV => 90.0 GeV /+ 0.5 GeV),
              (90.1 GeV => 91.9 GeV /+ 0.1 GeV),
              (92.0 GeV => 95.0 GeV /+ 0.5 GeV)) {
  integrate (bornproc)    { iterations = 2:1000:"gw", 1:2000 }
  record lineshape_born     (sqrts, integral (bornproc) / 1000)
  integrate (realcorr)    { iterations = 5:3000:"gw", 2:5000 }
  record lineshape_realcorr (sqrts, integral (realcorr) / 1000)
}

# Combine the plots to one graph
$title = "The Z Lineshape in $e^+e^-\to\mu^+\mu^-(\gamma)$"
graph g1 = lineshape_born & lineshape_realcorr { $draw_options = "withcolor blue" }

compile_analysis { $out_file = "Z-lineshape.dat" }
```

2 Second Day

2.1 Prerequisites

2.1.1 Phase Space and kinematic Observables

As you already know all possible states of a system correspond to specific point in phase space. Since particle physics is a relativistic theory its phase space is spanned by the 4-vectors of positions $Q = (t, \vec{x})$ and momenta $P = (E, \vec{p})$. However calculations turn out to be calculated most conveniently in momentum space. Therefor we define incoming $A_m(P_m)$ and outgoing $B_n(K_n)$ particles in a particular particle reaction. Of course all possible trajectories in particle physics are constraint by momentum and energy conservation.[13]

An arbitrary particle reaction can be written as

$$A_1(P_1), \ldots, A_m(P_m) \to B_1(K_1), \ldots, B_n(K_n) \quad . \tag{2.1}$$

In practice however only particle decays 2.2 and collisions 2.3 are considered:

$$A(P) \to B_1(K_1), \ldots, B_n(K_n) \tag{2.2}$$

$$A_1(P_1), A_2(P_2) \to B_1(K_1), \ldots, B_n(K_n) \tag{2.3}$$

Cross section σ and decay width Γ booth are Lorentz-invariant measures for probabilities of particle reaction rates and can be calculate from a corresponding matrix element $\mathcal{M}$.

The matrix element $\mathcal{M}$ itself contains all information corresponding to particular process. Therefor all formulas in this chapter for 2.2 and 2.3 are universal and valid for all particle reactions and do not depend on a particular theory.

According to *Fermis golden rule* we can calculate those probabilities knowing the phase space and a matrix element $\mathcal{M}$.

$$\mathrm{d}P(A_1, A_2, \ldots \to B_1, B_2, \ldots) = |\langle B_1, B_2, \ldots | A_1, A_2, \ldots \rangle|^2 \tag{2.4}$$

$$= (2\pi)^4 \delta^{(4)} \left(\sum_i P_i - \sum_i K_i \right) |\mathcal{M}|^2 \prod_i \frac{\mathrm{d}^3 \vec{k_i}}{(2\pi)^3 2E_i} \prod_j \frac{\mathrm{d}^3 \vec{p_j}}{(2\pi)^3 2E_j} \tag{2.5}$$

Note the delta-distribution $\delta^{(4)}$ reflects the overall momentum and energy conservation and the factor $(2\pi)^4$ comes from its normalization.

Decay rate Γ and cross section σ will both presented in their differential form since there total value can always be calculated by integrating over all variables.

In this course `WHIZARD` will be used to calculate the cross section σ since it is able to handle the integration of phase space and matrix element properly in a numerical way.

[13] `https://pdg.lbl.gov/2021/web/viewer.html?file=%2F2021/reviews/rpp2020-rev-kinematics.pdf`

Decay Width

In its differential form the decay rate Γ is given by

$$\mathrm{d}\Gamma(P, K_1, \dots) = \frac{(2\pi)^4}{2M} \cdot |\mathcal{M}|^2 \cdot \delta^{(4)} \left(P - \sum_i K_i \right) \prod_i \frac{\mathrm{d}^3 \vec{p_i}}{(2\pi)^3 2E_i} \quad . \tag{2.6}$$

The factor $2M$ can be obtained from the phase space by using 4-Momentum conservation $M^2 = P^2 = (K_1 + K_2)^2$ and some properties of the delta distribution. Note that M is the decaying particle's mass.

For a $1 \to 2$ in the decaying particle's rest frame one obtains

$$\mathrm{d}\Gamma(P, K) = \frac{1}{32\pi^2} \cdot |\mathcal{M}|^2 \cdot \frac{|\vec{k}|}{M^2} \mathrm{d}\cos\theta \mathrm{d}\varphi \quad . \tag{2.7}$$

Branching Ratios

For a decaying particle A and particular decay channel $A \to B_1, \dots, B_n$ often the so called *branching ratios* $\mathcal{B}$ are listed where

$$\mathcal{B} = \frac{\Gamma_{A \to B_1, \dots, B_n}}{\Gamma_{A \to X}} \quad . \tag{2.8}$$

In this notation X symbolizes the sum of all possible decay channels therefor $\mathcal{B} \in [0, 1]$.

Cross Section and Luminosity

Since it differs only in one more incoming particle in the phase space the cross section σ has a similar form to the decay rate. Again $\delta^{(4)}$ reflects the overall momentum and energy conservation and the different prefactor can be obtained in a similar way

$$\mathrm{d}\sigma(P_1, P_2, K_1, \dots) = \frac{(2\pi)^4}{\sqrt{(P_1 \cdot P_2)^2 - (m_1 m_2)^2}} \cdot |\mathcal{M}|^2 \cdot \delta^{(4)} \left(P_1 + P_2 - \sum_i K_i \right) \prod_i \frac{\mathrm{d}^3 \vec{p_i}}{(2\pi)^3 2E_i} \tag{2.9}$$

$$\approx \frac{(2\pi)^4}{4s} \cdot |\mathcal{M}|^2 \cdot \delta^{(4)} \left(P_1 + P_2 - \sum_i K_i \right) \prod_i \frac{\mathrm{d}^3 \vec{p_i}}{(2\pi)^3 2E_i} \quad . \tag{2.10}$$

Here $s = (P_1 + P_2)^2$ is one of the three so called *Mandelstam variables* which are a convenient set of Lorentz-invariant variables to parameterize the kinematic in $2 \to 2$ scattering processes. Note that $\sqrt{s}$ is approximately the same as the collider's energy E for $s \gg m_i^2$. In the center of mass frame (CMS)

$$\sqrt{s} = \sqrt{(P_1 + P_2)^2} \approx \sqrt{2\, P_1 \cdot P_2} = E_{\mathrm{CMS}} \quad . \tag{2.11}$$

For our propose it is furthermore reasonable to assume $s \gg m_i^2$. Therefor for a $2 \to 2$ scattering one obtains by using the δ-distribution

$$\mathrm{d}\sigma(P_1, P_2, K_1, K_2) = \frac{1}{64\pi^2 s} \cdot |\mathcal{M}|^2 \cdot \mathrm{d}\cos\theta \mathrm{d}\varphi \quad . \tag{2.12}$$

To calculate the number of events N to a certain type of interaction derived for the cross section σ in a real particle collider setting the luminosity L is also necessary. In contrast to the cross section which contain all information for particle reaction the luminosity reflects the properties of the particle collider like the collision rate of the initial state particles. Therefor

$$N = \int \mathrm{d}t L(t)\sigma(t) \quad . \tag{2.13}$$

Assuming nigher the cross section nor the luminosity having an explicit time t dependence

$$N = T \cdot L \cdot \sigma \tag{2.14}$$

where T is the running time of the collider.

Angular Distribution

The distribution for any (kinematic) observable can be derived from 2.6 or 2.12 by elementary differential operations and integrating out the other dependencies.

One of them is the angular distribution. Due to the conservation of energy and momentum, the only nontrivial distribution for a $2 \to 2$ process is the distribution of the scattering angle θ, the angle between a final state particle and the beam axis. Therefor

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} = \frac{\sin\theta}{32\pi s} \cdot |\mathcal{M}|^2 \quad . \tag{2.15}$$

For further information on the kinematics on particle colliders see [8].

2.1.2 The Electroweak Interaction and its Feynman Rules

As mentioned in 1.1.1 one of the fundamental forces of nature is the *electroweak interaction*. In a simplistic way it can be understood as a unification of the *electromagnetic* and the *weak* forces.

In the Standard Model this unification is derived by the so called *Higgs-mechanism* which also generates the masses for all particles.

In contrast to the electromagnetic interaction which is mediated by the massless photon γ, the weak interaction is mediated by the massive vector bosons $W^\pm$ and Z and the massive *Higgs*-boson h.

The electroweak interaction conserves the electromagnetic charge as well as the lepton number both in total and for each generation separately. The charges and lepton numbers of all leptons and the electroweak mediators can be found in table 2.

Particle	C	L_e	L_μ	L_τ
$e^\pm$	±1	∓1	0	0
$\mu^\pm$	±1	0	∓1	0
$\tau^\pm$	±1	0	0	∓1
$\nu_e/\bar{\nu}_e$	0	∓1	0	0
$\nu_\mu/\bar{\nu}_\mu$	0	0	∓1	0
$\nu_\tau/\bar{\nu}_\tau$	0	0	0	∓1
$W^\pm$	±1	0	0	0
Z,h,γ	0	0	0	0

Table 2: Information about the quantum numbers of all lepton and the mediators of the electroweak interaction. C is the charge and L_i is the lepton number in each generation.

2.1.3 Feynman Diagrams and Perturbation Theory

Even if it is usually not possible to calculate the matrix element of an interacting theory exactly, one can approach the solution pertubatively. This is where the so-called *Feynman rules* of the theory come into play. Each Feynman diagram is a pictorial representation of a mathematical expression contributing to the matrix element. It is a intuitive way to get all contributions to certain order of perturbation theory for a particular process.

There are tree types of elements a Feynman diagram can be build form. First there are vertices. They can be understood as an interaction of several particles at a point in spacetime. Each of them is proportional to an order of the coupling constant of the theory it was derived from. For this course we oversimplify it by calling the coupling constant just α_{EW}. In reality there is more than one coupling constant involved and one has to be careful to get all contributions to obtain a certain precision.

The electroweak interaction involves vertices shown in figure 2.

The different line styles involving bosons have no meaning other than to distinguish vector bosons from the scalar Higgs h. However for the fermion the direction of the arrow distinguish a fermion from its anti-fermion what becomes especially important when momentum is assigned to the particles.

Using table 2 you might check that each of those vertices conserves charge C the lepton number in each generation L_i. This not happened per accident. It is an important feature of the Feynman rule expansion. Therefor it's obvious that diagrams build from these vertices also conserves C and L_i.

The other sets of expressions you need to know to calculate a matrix element are rules for the lines themselves. An internal line is called propagator. The lines corresponding to a final or initial state particle are called a external lines have different expressions then the propagators. For this course we won't go any deeper in that.

For building a diagram corresponding to a correction of a certain order in perturbation theory you just need to put two ends with the same particle together. The initial and final state

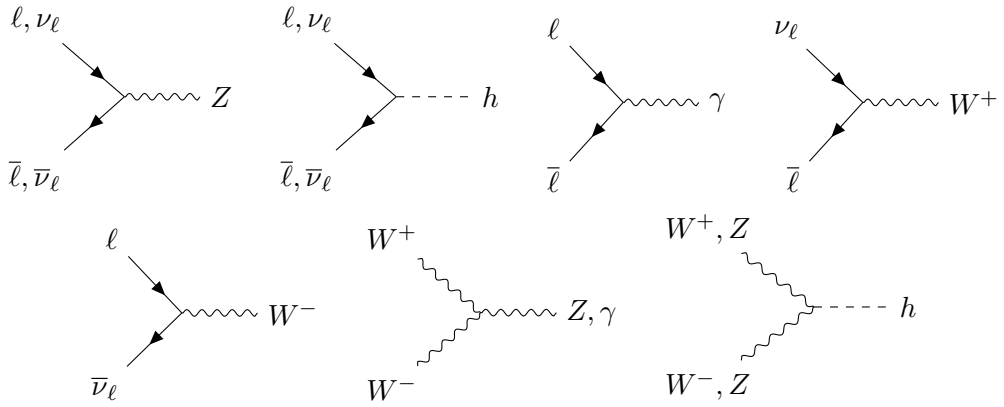


Figure 2: Some vertices of the electroweak interaction. Note that there are also vertices coupling four bosons to each other but for now we ignore them. You might find them on your own using table 2.

particles are the left and right sided loose ends of the diagram.

An exemplary contribution to the matrix element of $e^+e^- \to W^+W^-$ is shown in figure 3. Of course there a several more contributing diagrams to this process. You might find the other diagrams yourself.[14]

Of course you can draw diagrams with more than two vertices but they would contribute to a different order in perturbation theory. For now just say the order in perturbation theory corresponds to the number of vertices of the diagram. However in a real calculation it's different since for example a vertex itself might have a higher order in the coupling constant.

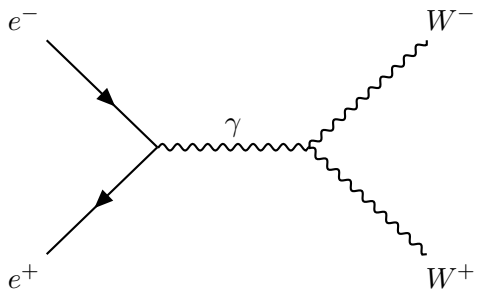


Figure 3: One of the Feynman diagrams contributing to the matrix element of $e^+e^- \to W^+W^-$.

[14] If you want to draw Feynman diagram in your protocol see the TIKZ page for latex [9].

2.2 Exercises

Now the goal is to analysis

$$e^+e^- \to W^+W^- \to \ell_1\ell_2\ell_3\ell_4, \quad \ell_i \in \{e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau\} \quad . \tag{2.16}$$

Therefor you will simulate two different approximations of the process 2.16:

- For this analysis you can assume an idealized detector that can detect all final state particles, including neutrinos, under all circumstances. Therefor there are no blind spots and all non-identical particles can be distinguished from each other.
- Due to the high collision energy $\sqrt{s}$ leptons masses can safely be neglected since $m_\ell \ll \sqrt{s}$.
- You will find the newest values of masses m and widths w [15] of $W^\pm$, Z and H in the particle data book (PDG)[16].

Before calculating any process think about what subset final state leptons are possible from a decaying W-Boson pair. Choose your final leptons accordingly for each task.

B1. **Sub-Process-Integration**: First consider $e^+e^- \to W^+W^- \to \ell_1\ell_2\ell_3\ell_4$ and use `WHIZARD` to *scan* over the process energy $\sqrt{s}$ again. As a simplification use the branching ratios for W^+ and W^- from the PDB instead of calculating the full decays. Adjust the lower bound to the intermediate state. The upper bound should be $\sqrt{s} = 4 \cdot M_W$. The chosen energy steps should provide an insight about all relevant phenomena of the process. Discuss your results.

B2. **Hard-Process-Integration**: Now consider the $e^+e^- \to \ell_1\ell_2\ell_3\ell_4$ and use `WHIZARD` to *scan* over the process energy $\sqrt{s}$. Choose the upper and lower bound the same as the previous task. Again the chosen energy steps should provide an insight about all relevant phenomena of the process. To obtain a finite results it is necessary to apply a cut on the kinematics. For this use $|\cos\theta| < 0.985$. Compare your results with the previous task.

B3. **Angular Distribution**: Now consider $e^+e^- \to W^+W^-$ and use `WHIZARD`'s `simulate` command to produce the θ-distribution of W^+ and W^- with $\theta \in [0°, 180°]$. Use `WHIZARD` to generate a histogram[17] of this distribution. The histogram should be normalized to the total cross section of the process. For this use `$sample_normalization="sigma/N"`. A bin width of 20° will be sufficient. $\sqrt{s}$ should be the about the energy of the cross sections maximum from task B2.. If you don't have produced enough events on the first run you can combine the data of several histograms.

[15] A unstable particle in scattering process shows up as resonance in the cross section what can be estimated via a relativistic generalization of the *Breit-Wigner*-Formula. Therefor near the resonance $\mathcal{M} \sim (P^2 - m^2 + imw)^{-1}$. For this distribution the width can be derived.

[16] `https://pdg.lbl.gov/2021/download/db2020.pdf`

[17] An example how to generate a histogram can in be found in `W-endpoint.sin` which is located in the `examples` folder.

Refinements

If all prior tasks are completed and there is still enough time you might proceed with the following tasks. Nevertheless proceeding won't give you some additional or hidden bonus points. Therefor it's possible achieve the highest grade in the course without doing any of these.

R1. **Angular Cuts**: Redo task B1. and B2. under the assumption that no particles could be detected within a range of $\theta_b = 20°$ around the beam axis. For that you need to set a `cut` on the matrix element. Compare your results with the results of the previous tasks.

R2. **Background**: Do an analysis of the background of the process $e^+e^- \to W^+W^- \to \ell_1\ell_2\ell_3\ell_4$. For this list all two boson intermediate states. Carefully think which can be neglected. Compare your results. Redo the tasks B1. and B3. for the remaining processes.

Report

- Document the *seeds* used for each run with `WHIZARD` and write it in the description of each of your plots.
- Your protocol has to contain your `sindarin` files as well as your full `WHIZARD` data as an appendix.
- Your `sindarin` files are part of your protocol, so make sure that they are well structured and commented in a way that helps understanding them.
- Make sure hat your plots have proper x- and y-axis labels as well as a reasonable title.
- Describe your observation. So they match your expectations?
- Discuss the error values of your simulations. Do their values seam reasonable?

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